Instrumental Variable Estimation 1: Framework

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Last updated: 2021-05-18

Introduction

Introduction: Endogeneity Problem and its Solution

- This chapter introduces **instrumental variable (IV, 操作変数)** method as a solution to endogeneity problem.
- The lecture plan
  1. More on endogeneity issues
  2. Framework
  3. Implementation in R

Endogeneity

More Examples of Endogeneity Problem

- Talk more on endogeneity problems.
  1. Omitted variable bias
  2. Measurement error
  3. Simultaneity

Example 1: More on Omitted Variable Bias

- Remember the wage regression equation (true model)

$$\begin{aligned} \log W_i &= \beta_0 + \beta_1 S_i + \beta_2 A_i + u_i \\ E[u_i | S_i, A_i] &= 0 \end{aligned}$$

where W_i is wage, S_i is the years of schooling, and A_i is the ability.

- Suppose that you omit A_i and run the following regression instead.

$$\log W_i = alpha_0 + \alpha_1 S_i + v_i$$

Notice that $v_i = \beta_2 A_i + u_i$, so that S_i and v_i is likely to be correlated.

Does addition control variables solve OVB?

- You might want to add more and more additional variables to capture the effect of ability.
  - Test scores, GPA, SAT scores, etc...

- However, how many variables should we conditioning so that S_i is indeed exogenous?

- Multivariate regression is valid under **selection on observables**.

- If there exists **unobserved heterogeneity** that matters both in wage and years of schooling, multivariate regression cannot deal with OVB.

Example 2: Measurement error

- Reporting error, wrong understanding of the question, etc...
- Consider the regression

$$y_i = \beta_0 + \beta_1 x_i^* + \epsilon_i$$

- Here, we only observe x_i with error:

$$x_i = x_i^* + e_i$$

  - e_i is independent from ϵ_i and x_i^* (called classical measurement error)
  - You can think e_i as a noise added to the data.

- The regression equation is

$$\begin{aligned} y_i = & \ \beta_0 + \beta_1(x_i - e_i) + \epsilon_i \\ = & \ \beta_0 + \beta_1 x_i + (\epsilon_i - \beta_1 e_i) \end{aligned}$$

- Then we have correlation between x_i and the error $\epsilon_i - \beta_1 e_i$, violating the mean independence assumption.
- Measurement error in explantory variables leads to smaller estimates, known as **attenuation bias**.

Example 3: Simultaneity (同時性) or reverse causality

- **Dependent and explanatory variable are determined simultaneously.**
- Consider the demand and supply curve

$$q^d = \beta_0^d + \beta_1^d p + \beta_2^d x + u^d$$
$$q^s = \beta_0^s + \beta_1^s p + \beta_2^s z + u^s$$

- The equilibrium price and quantity are determined by $q^d = q^s$.

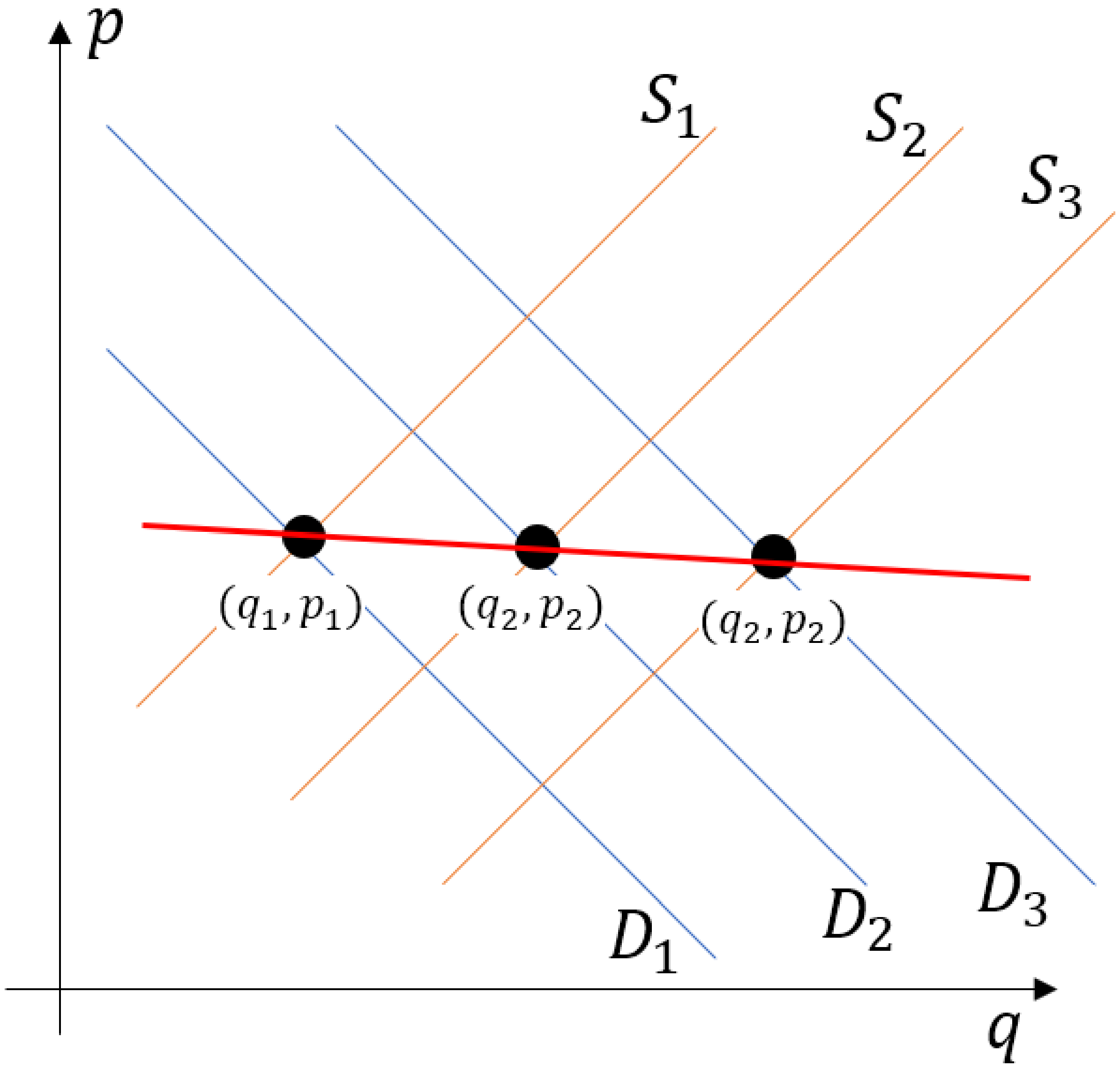
- In this case,

$$p = \frac{(\beta_2^s z - \beta_2^d z) + (\beta_0^s - \beta_0^d) + (u^s - u^d)}{\beta_1^d - \beta_1^s}$$

  implying the correlation between the price and the error term.

- Putting this differently, the data points we observed is the intersection of these supply and demand curves.

- How can we distinguish demand and supply?



Causal Effect of Price on Quantity?

- What is a causal effect of price on quantity? What is the sign (if it exists)?
- For consumer: Higher prices leads to lower demand.
- For (price-taking) firms: Higher prices leads to more supply.
- "Causal effect of price on quantity" is a meaningless concept unless you specify either demand or supply.

IV Idea

Idea of IV Regression

- Let's start with a simple case.

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, Cov(x_i, \epsilon_i) \neq 0$$

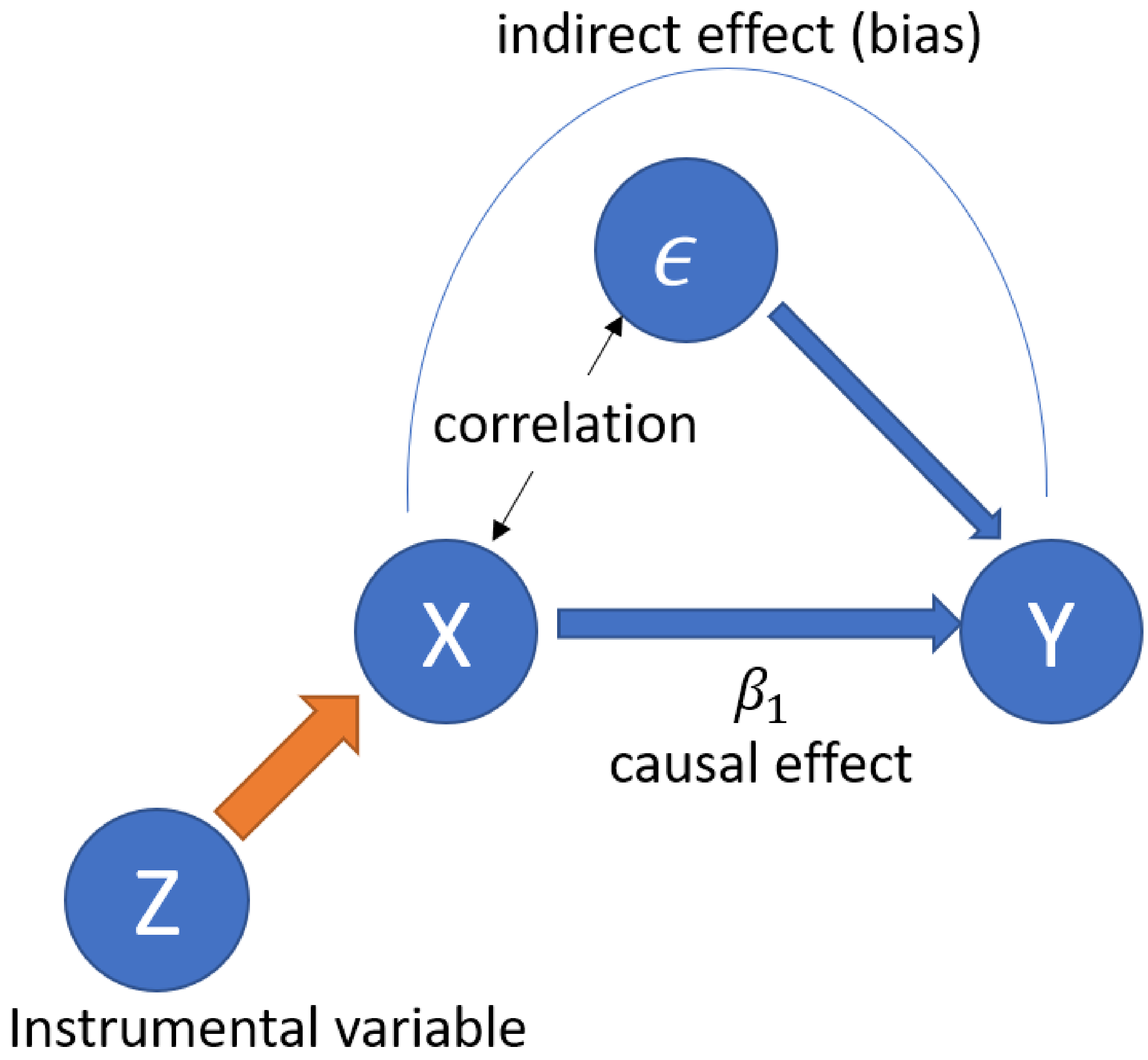
- Define the variable z_i named **instrumental variable (IV)** that satisfies the following conditions:

  1. **Independence (独立性)**: $Cov(z_i, \epsilon_i) = 0$. No correlation between IV and error.
  2. **Relevance (関連性)**: $Cov(z_i, x_i) \neq 0$. Correlation between IV and endogenous variable x_i.

- Idea: Use the variation of x_i **induced by instrument** z_i to estimate the direct (causal) effect of x_i on y_i, that is β_1!.

More on Idea

1. Intuitively, the OLS estimator captures the correlation between x and y.
2. If there is no correlation between x and ϵ, it captures the causal effect β_1.
3. If not, the OLS estimator captures both direct and indirect effect, the latter of which is bias.
4. Now, let's capture the variation of x due to instrument z,
   - Such a variation should exist under **relevance** assumption.
   - Such a variation should not be correlated with the error under **independence assumption**
5. By looking at the correlation between such variation and y, you can get the causal effect β_1.


indirect effect (bias)
ϵ
correlation
X
Y
β_1
causal effect
Z
Instrumental variable

Identification of Parameter with IV

- Taking the covariance between y_i and z_i

$$Cov(y_i, z_i) = \beta_1 Cov(x_i, z_i) + Cov(\epsilon_i, z_i)$$

- IV conditions implie that

$$\beta_1 = \frac{Cov(y_i, z_i)}{Cov(x_i, z_i)}$$

- Question: Can you see the roles of IV conditions?

IV General Framework

Multiple endogenous variables and instruments

- Consider the model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_K X_{Ki} + \beta_{K+1} W_{1i} + \cdots + \beta_{K+R} W_{Ri} + u_i,$$

  - Y_i is the dependent variable
  - $X_{1i}, \ldots, X_{Ki}$ are K endogenous (内生) regressors: $Cov(X_{ki}, u_i) \neq 0$ for all k.
  - $W_{1i}, \ldots, W_{Ri}$ are R exogenous (外生) regressors which are uncorrelated with u_i. $Cov(W_{ri}, u_i) = 0$ for all r.
  - u_i is the error term
  - $Z_{1i}, \ldots, Z_{Mi}$ are M instrumental variables
  - $\beta_0, \ldots, \beta_{K+R}$ are $1 + K + R$ unknown regression coefficients

Two Stage Least Squares (2SLS, 二段階最小二乗法)

- Step 1: **First-stage regression(s)**
  - For each of the endogenous variables ($X_{1i}, \ldots, X_{ki}$), run an OLS regression on all IVs ($Z_{1i}, \ldots, Z_{mi}$), all exogenous variables ($W_{1i}, \ldots, W_{ri}$) and an intercept.
  - Compute the fitted values ($\widehat{X}_{1i}, \ldots, \widehat{X}_{ki}$).

- Step 2: **Second-stage regression**
  - Regress the dependent variable Y_i on **the predicted values** of all endogenous regressors ($\widehat{X}_{1i}, \ldots, \widehat{X}_{ki}$), all exogenous variables ($W_{1i}, \ldots, W_{ri}$) and an intercept using OLS.
  - This gives $\widehat{\beta}_0^{TSLS}, \ldots, \widehat{\beta}_{k+r}^{TSLS}$, the 2SLS estimates of the model coefficients.

Why 2SLS works

- Consider a simple case with 1 endogenous variable and 1 IV.
- In the first stage, we estimate

$$x_i = \pi_0 + \pi_1 z_i + v_i$$

  by OLS and obtain the fitted value $\widehat{x}_i = \widehat{\pi}_0 + \widehat{\pi}_1 z_i$.
- In the second stage, we estimate

$$y_i = \beta_0 + \beta_1 \widehat{x}_i + u_i$$

- Since $\widehat{x}_i$ depends only on z_i, which is uncorrelated with u_i, the second stage can estimate β_1 without bias.

Conditions for IV

Conditions for Valid IVs: Necessary condition

- Depending on the number of IVs, we have three cases
  1. Over-identification (過剰識別): $M > K$
  2. Just identification (丁度識別): $M = K$
  3. Under-identification (過小識別): $M < K$
- The necessary condition is $M \geq K$.
  - We should have more IVs than endogenous variables.

Sufficient condition

- In a general framework, the sufficient conditions for valid instruments
    1. **Independence**: $Cov(Z_{mi}, u_i) = 0$ for all m.
    2. **Relevance**: In the second stage regression, the variables

$$\left(\widehat{X}_{1i}, \ldots, \widehat{X}_{ki}, W_{1i}, \ldots, W_{ri}, 1\right)$$

       are **not perfectly multicollinear**.

- What does the relevance condition mean?

Relevance condition

- In the simple example above, The first stage is

$$x_i = \pi_0 + \pi_1 z_i + v_i$$

  and the second stage is

$$y_i = \beta_0 + \beta_1 \widehat{x}_i + u_i$$

- The second stage would have perfect multicollinarity if $\pi_1 = 0$ (i.e., $\widehat{x}_i = \pi_0$).

- Back to the general case, the first stage for X_k can be written as

$$X_{ki} = \pi_0 + \pi_1 Z_{1i} + \cdots + \pi_M Z_{Mi} + \pi_{M+1} W_{1i} + \cdots + \pi_{M+R} W_{Ri}$$

  and one of $\pi_1, \cdots, \pi_M$ should be non-zero.

- Intuitively speaking, **the instruments should be correlated with endogenous variables after controlling for exogenous variables**

Check Instrument Validity: Relevance

- **Weak instruments (弱操作変数)** if those instruments explain little variation in the endogenous variables.

- Weak instruments lead to
  1. imprecise estimates (higher standard errors)
  2. The asymptotic distribution would deviate from a normal distribution even if we have a large sample.

A rule of thumb to check the relevance conditions.

- Consider the case with one endogenous variable X_{1i}.
- The first stage regression

$$X_k = \pi_0 + \pi_1 Z_{1i} + \cdots + \pi_M Z_{Mi} + \pi_{M+1} W_{1i} + \cdots + \pi_{M+R} W_{Ri}$$

- Conduct F test.

$$\begin{aligned} H_0 &: \pi_1 = \cdots = \pi_M = 0 \\ H_1 &: otherwise \end{aligned}$$

- If we can reject this, we can say no concern for weak instruments.

- A rule of thumbs is that the F statistic should be larger than 10. (Stock, Wright, and Yogo 2012)

Independence (Instrument exogeneity)

- This is essentially non-testable assumption, as in mean independence assumption in OLS.
- Justification of this assumption depends on a context, institutional features, etc...