# Regression Discontinuity 2: Implementation in R

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# **Close election design**

#### **Question and Estimation strategy**

- Lee, D.S., Moretti, E., and M. Butler, 2004, Do Voters Affect or Elect Policies? Evidence from the U.S. House, Quarterly Journal of Economics 119, 807-859.
- Do voters affect policy itself or do they just select politician?
- The roll-call voting record  $RC_t$  of the representative in the district following the election t can be written as

$$RC_t = (1 - D_t)y_t + D_t x_t,$$

 $D_t$ : indicator variable for whether the Democrat won election t
  $x_t$  ( $y_t$ ): the policy implemented by the Democrat (the Repulican) at t

• Under some conditions, it can be expressed as

$$RC_t = constant + \pi_0 P_t^* + \pi_1 D_t + \epsilon_t$$
 (1)  
 $RC_{t+1} = constant + \pi_0 P_{t+1}^* + \pi_1 D_{t+1} + \epsilon_{t+1}$  (2)

 $\circ P_t^*$ : voters' underlying popularity (the electoral strength) of the Democrat. It is defined as the probability that party D will win if parties D and R are expected to choose their blis points, not moderating points.

- What we try to know is whether  $\pi_0 = 0$  or  $\pi_1 = 0$ , or neither, meaning what affect representative's roll-call voting, in other words, politician's decision.
  - If  $\pi_1 = 0$ , the roll-call voting of the representative in the district does not vary regarless of who wins (called **Complete Convergence**). That is both parties choose the exactly same policy. The policy position is determined only by the voter's underlying popularity.
  - If  $\pi_0 = 0$ , the roll-call voting of the representative in the district does not affected by voters' underlying popularity (called **Complete Divergence**). This can be interpretted that voters can not affect policy, but merely elect politicians' fixed policies.
  - If else, both parties select different policies, but voters can affect policy (called **Partial Convergence**).

- The problem is we cannot estimate equations (1) and (2), because we cannot observe  $P_t^*$ .
- This brings **two issues** to figure out in order to identify  $\pi_0$  and  $\pi_1$ .
  - 1. Simple comparison of  $RC_t$  between  $D_t = 1$  and  $D_t = 0$  without controlling on  $P_t^*$  leads endogeneity bias, since  $P_t^*$  tends to be higher among  $D_t = 1$ .  $\Rightarrow$  We need to somehow control  $P_t^* \Rightarrow \mathsf{RDD}$ 
    - By focusing on close elections (when voteshares of both parties are very tight), we can compare the cases between when  $D_t = 1$  and  $D_t = 0$ , fixing  $P_t^*$  constant.  $\Rightarrow$  Being able to identify  $\pi_1$ .
  - 2. Because  $P_t^*$  is directly unobservable, we have to somehow find variation of  $P_t^*$  to identify  $\pi_0$ .
    - $\Rightarrow$  Incumbency advantage
      - The random assignment of who wins in the first election generates random assignment in which candidate has greater electoral strength for the next election.
         This requires two period analysis.

#### Identification

• The conditional expectation of equation (2) is:

$$egin{aligned} \lim_{v \downarrow 0.5} E[RC_{t+1} | V_t = v] &= constant + \pi_0 E[P_{t+1}^* | D_t = 1, V_t = 0.5] \ &+ \pi_1 E[D_{t+1} | D_t = 1, V_t = 0.5] \ &= constant + \pi_0 P_{t+1}^{*D} + \pi_1 P_{t+1}^D \ & ext{lim} \ E[RC_{t+1} | V_t = v] = constant + \pi_0 E[P_{t+1}^* | D_t = 0, V_t = 0.5] \ &+ \pi_1 E[D_{t+1} | D_t = 0, V_t = 0.5] \ &= constant + \pi_0 P_{t+1}^{*R} + \pi_1 P_{t+1}^R \end{aligned}$$

- $\circ V_t$  is voteshare of the Democrat in election t, and threshold is 0.5.
- $\circ P_{t+1}^{*D} \equiv E[P_{t+1}^* | D_t = 1, V_t = 0.5], P_{t+1}^{*R} \equiv E[P_{t+1}^* | D_t = 0, V_t = 0.5]$
- $P_{t+1}^{D}$  ( $P_{t+1}^{R}$ ) is equilibrium probability that Democrat wins in election t+1 when Democrat (Republican) won in election t.

#### **Estimation**

• When one could randomize  $D_t$  by restricting data close to the threshold,

$$\underbrace{E[RC_{t+1}|D_{t}=1] - E[RC_{t+1}|D_{t}=0]}_{\text{Observable}} = \pi_{0}(P_{t+1}^{*D} - P_{t+1}^{*R}) + \pi_{1}(P_{t+1}^{D} - P_{t+1}^{R})$$

$$\equiv \underbrace{\gamma}_{\text{Total effect of initial win on future roll call votes}}_{\text{Total effect of initial win on future roll call votes}}$$

$$\underbrace{E[RC_{t}|D_{t}=1] - E[RC_{t}|D_{t}=0]}_{\text{Observable}} = \pi_{1} \quad (4)$$

$$\underbrace{E[D_{t+1}|D_{t}=1] - E[D_{t+1}|D_{t}=0]}_{\text{Observable}} = P_{t+1}^{D} - P_{t+1}^{R} \quad (5)$$

• Therefore,  $\pi_0(P_{t+1}^{*D}-P_{t+1}^{*R})$  can be estimated by  $\gamma-\pi_1(P_{t+1}^D-P_{t+1}^R)$ 

#### Data

- There are two main data sets in this project.
- The first is a measure of how liberal an official voted, broght from ADA score for 1946–1995.
   ADA varies from 0 to 100 for each representative. Higher scores correspond to a more "liberal" voting record.
- The running variable in this study is the vote share. That is the share of all votes that went to a Democrat across Congressional districts.
  - U. S. House elections are held every two years.

- **Panel data** (1946–1995  $\times$  all districs around the U.S.)
- Main variables
  - $\circ$  score: ADA score in Congressional session t of the representative elected at k  $(RC_t)$
  - $\circ$  democrat: indicator whether the Democrat wins in election t  $(D_t)$
  - $\circ$  lagdemocrat: indicator whether the Democrat wins in election t-1  $(D_{t-1})$
  - $\circ$  demvoteshare: voteshare at district k in election t  $(V_t)$
  - $\circ$  lagdemvoteshare: voteshare at district k in the previous election, t-1  $(V_{t-1})$
- For example, one specific row of the dataset has the voteshares and the results of the November 1992 election (period t) and the November 1990 election (period t-1) at district k, and the ADA score of 1993–1994 Congressional session (period t).

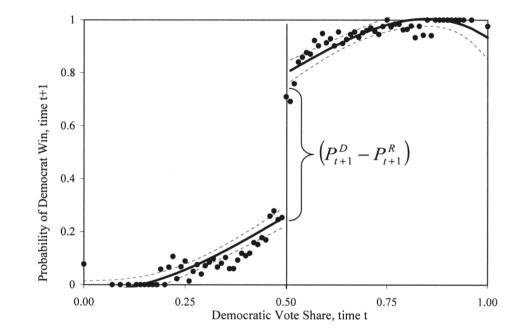
# **Graphical Analysis**

## **Graphical Analysis**

- Results of the analysis will be seen later. Here, we learn how to implement graphical analysis first.
- RD analyses hinge on their graphical analyses.
- always start with visual inspection to check which model (e.g. linear or nonlinear) is plausible.

#### Outcomes by the running variables

• First, we try to create this figure from the article.



- The dependent variable is probability of Democrat victory in election t+1 and the independent is voteshare in election t.
- Then, we will see what happens when we change bandwidth and functional form.

```
library(tidyverse)
library(haven)
library(estimatr)
library(texreg)
library(latex2exp)
# Download data
read_data <- function(df)</pre>
ł
  full_path <- paste("https://raw.github.com/scunning1975/mixtape/master/",</pre>
                       df, sep = "")
  df <- read_dta(full_path)</pre>
  return(df)
}
lmb_data <- read_data("lmb-data.dta")</pre>
```

• First, you have more than 10,000 data points, so reduce them for scatter plot.

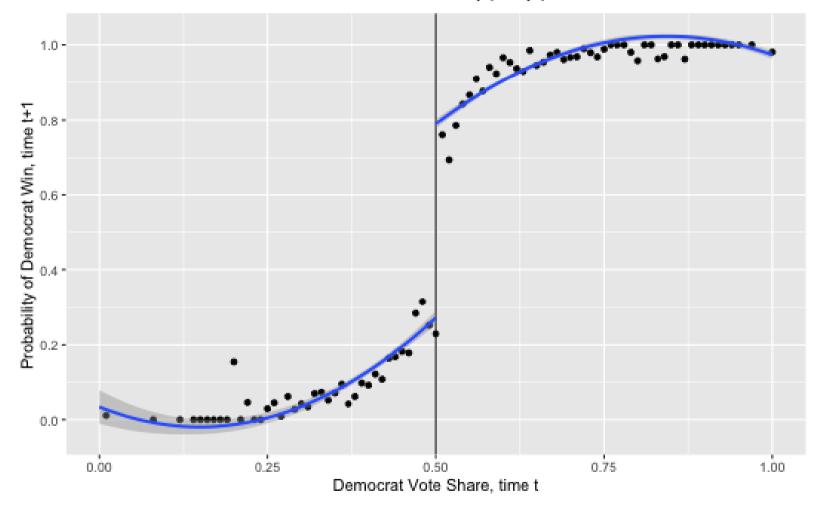
```
#aggregating the data
# calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_data$democrat, cut(lmb_data$lagdemvoteshare, 100)) %>%
    lapply(mean) %>%
    unlist()
#createing new data frame for plotting
```

```
agg_lmb_data <- data.frame(democrat = demmeans, lagdemvoteshare = seq(0.01,1, by = 0.01))</pre>
```

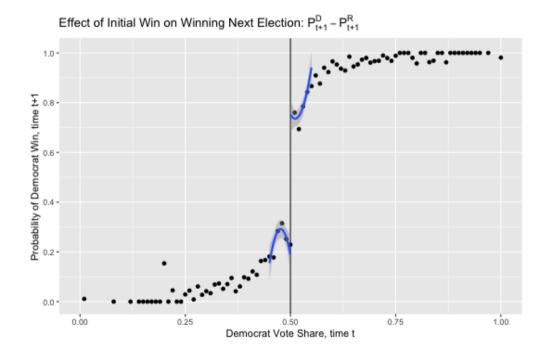
## Quadratic fitting in all data

```
#grouping above or below threshold
lmb data <- lmb data %>%
 mutate(gg_group = if_else(lagdemvoteshare > 0.5, 1,0))
#plotting
gg srd = ggplot(data=lmb data, aes(lagdemvoteshare, democrat)) +
    geom point(aes(x = lagdemvoteshare, y = democrat), data = agg lmb data) +
   xlim(0,1) + ylim(-0.1,1.1) +
   geom vline(xintercept = 0.5) +
   xlab("Democrat Vote Share, time t") +
   ylab("Probability of Democrat Win, time t+1") +
    scale y continuous(breaks=seg(0,1,0.2)) +
    ggtitle(TeX("Effect of Initial Win on Winning Next Election: \V = P^R \{t+1\}
gg_srd + stat_smooth(aes(lagdemvoteshare, democrat, group = gg_group),
                    method = "lm", formula = y \sim x + I(x^2))
```

Effect of Initial Win on Winning Next Election:  $P_{t+1}^{D} - P_{t+1}^{R}$ 



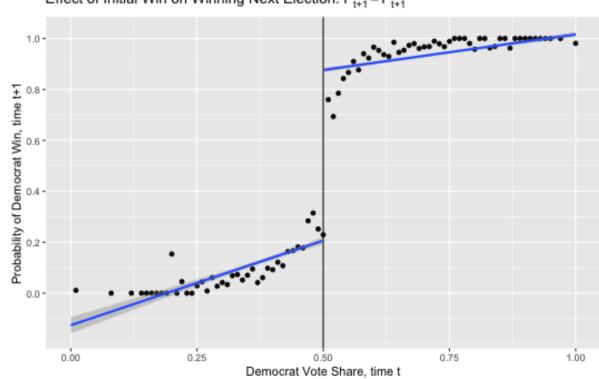
#### Quadratic fitting; limited to +/- 0.05



• Notice that confidence interval widens. But, lines fit plots better.

#### Linear different slops

gg\_srd + stat\_smooth(aes(lagdemvoteshare, democrat, group = gg\_group), method = "lm")

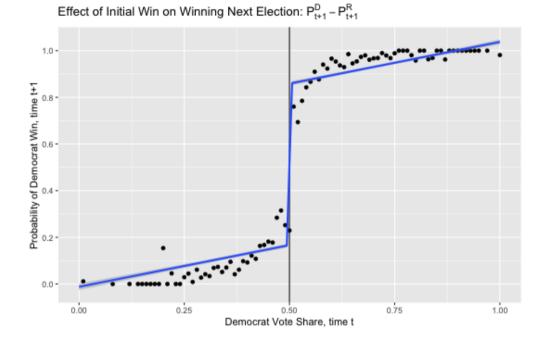


Effect of Initial Win on Winning Next Election:  $P_{t+1}^{D} - P_{t+1}^{R}$ 

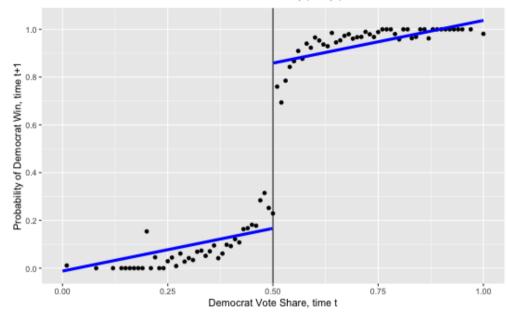
#### Linear common slop

• Alternatively, this can avoid showing line across the threshold.

```
lm_tmp <- lm(democrat ~ lagdemvoteshare + I(lagdemvoteshare>0.5), data = lmb_data)
lm_fun <- function(x) predict(lm_tmp, data.frame(lagdemvoteshare = x)) #output is predicted democray
gg_srd +
stat_function(
    data = data.frame(x = c(0, 1), y = c(0, 1)), aes(x = x, y=y),
    fun = lm_fun, xlim = c(0, 0.499),
    col="blue", size = 1.5) +
stat_function(
    data = data.frame(x = c(0, 1), y = c(0, 1)), aes(x = x, y=y),
    fun = lm_fun, xlim = c(0.501, 1),
    col="blue", size = 1.5)</pre>
```

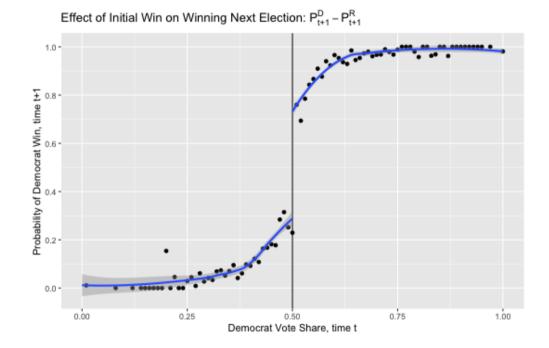


Effect of Initial Win on Winning Next Election:  $P_{t+1}^{D} - P_{t+1}^{R}$ 



### Loess fitting

gg\_srd + stat\_smooth(aes(lagdemvoteshare, democrat, group = gg\_group), method = "loess")

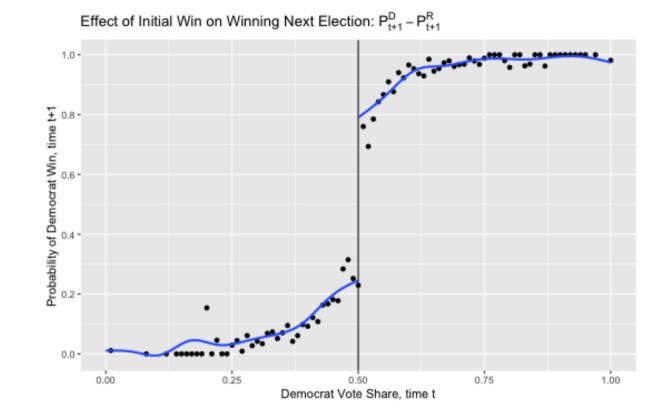


• Compared to the quadratic case, variance gets bigger but the prediction fits the points better.

## Kernel-weighted local polynomial regressions

```
library(stats)
```

```
smooth dem0 <- lmb data %>%
 filter(lagdemvoteshare < 0.5) %>%
 dplyr::select(democrat, lagdemvoteshare) %>%
 na.omit()
smooth_dem0 <- as_tibble(ksmooth(smooth_dem0$lagdemvoteshare, smooth_dem0$democrat,</pre>
                                 kernel = "box", bandwidth = 0.1))
smooth dem1 <- lmb data %>%
 filter(lagdemvoteshare >= 0.5) %>%
 dplyr::select(democrat, lagdemvoteshare) %>%
 na.omit()
smooth_dem1 <- as_tibble(ksmooth(smooth_dem1$lagdemvoteshare, smooth_dem1$democrat,</pre>
                                 kernel = "box", bandwidth = 0.1))
gg_srd +
  geom_smooth(aes(x, y), data = smooth_dem0) +
  geom_smooth(aes(x, y), data = smooth_dem1)
```



#### Model and Bandwidth selection - bias-variance tradeoff

- How should we pick the "right" model and bandwidth?
- There's always a **trade-off between bias and variance** when choosing bandwidth and polynomial length.
  - Bias: distance between your prediction and true value
  - Variance: width of your prediction
  - The shorter the window and the more flexible (e.g. higher-order polynomials) the model, the lower the bias, but because you have less data, the variance in your estimate increases.
- Always, it's important to show robustness.

- Model selection
  - Higher-order polynomials can lead to overfitting (Gelman and Imbens 2019). They recommend using local linear regressions with linear and quadratic forms only.
  - Local linear regression with a kernel smoother is a popular choice
- Bandwidth selection:
  - Optimal bandwidth selection: Imbens and Kalyanaraman (2011), Calonico, Cattaneo, and Titiunik (2014) implimentation will be at the last slide
  - Cross validation: Imbens and Lemieux (2008)

# **Quantitative analysis**

### Quantitative analysis

• Our next goal is to replicate the quantitaive results of Lee, Moretti, and Butler (2004) in the table below.

	$\gamma$	$\pi_1$	$P^{D}_{t+1} - P^{R}_{t+}$	$_{1}\pi_{1}(P^{D}_{t+1}$ –	$-P^R_{t+1})$	$\pi_0(P_{t+1}^{*D} -$	$P_{t+1}^{*R}$
Variable	$ADA_{t+1}$	$ADA_t$	$DEM_{t+1}$				
Estimated gap	21.2 (1.9)	47.6 (1.3)	0.48 (0.02)				
				22.84 (2.2)		-1.64 (2.0)	

- The analysis restrics only observations where the Democrat voteshare is between 48 percent and 52 percent, so that the number of observations is 915.
- From the second column, complete convergence is rejected.
- The last column of the statistical insignificance shows that voters primarily elect policies rather than affect policies.
- **Complete divergence** is supported by this analysis.

```
# Restrict data containg the Democrat vote share between 48 percent and 52 percent
# `lagdemvoteshare` is the Dem. voteshare of the t-1 period
lmb subset <- lmb data %>%
  filter(lagdemvoteshare>.48 & lagdemvoteshare<.52)</pre>
\# E[ADA_{t+1}] = \gamma
lm_1 <- lm_robust(score ~ lagdemocrat, data = lmb_subset, se_type = "HC1")</pre>
\# E[ADA_{t}] = pi_1
lm 2 <- lm robust(score ~ democrat, data = lmb_subset, se_type = "HC1")</pre>
\# E[D_{t+1}] = P_{t+1}^{-} - P_{t+1}^{-}
lm 3 <- lm robust(democrat ~ lagdemocrat, data = lmb subset, se type = "HC1")</pre>
screenreg(l = list(lm_1, lm_2, lm_3),
          digits = 2,
          # caption = 'title',
          custom.model.names = c("ADA t+1", "ADA t", "DEM t+1"),
          include.ci = F.
          include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
          include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
          custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
          # select coefficients to report
          stars = numeric(0))
```

##				
##	=============	=========	=========	=======
##		ADA_t+1	ADA_t	DEM_t+1
##				
##	lagdemocrat	21.28		0.48
##		(1.95)		(0.03)
##	democrat		47.71	
##			(1.36)	
##				
##	Num. obs.	915	915	915
##	============	========	=========	=======

- The results are slightly different. But ignore that for now.
- From now on, we will see how the results depend on **bandwidth** and **fanctional form**.

#### Same specification in all the data

#using all data (note data used is lmb\_data, not lmb\_subset)

```
lm_1 <- lm_robust(score ~ lagdemocrat, data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat, data = lmb_data, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat, data = lmb_data, se_type = "HC1")</pre>
```

```
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```

##				
##	============	===========	============	==========
##		ADA_t+1	ADA_t	DEM_t+1
##				
##	lagdemocrat	31.51		0.82
##		(0.48)		(0.01)
##	democrat		40.76	
##			(0.42)	
##				
##	Num. obs.	13588	13588	13588
##	============	============	============	=========

• Here we see that simply running the regression yields different estimates when we include data far from the cutoff itself.

#### Controls for the running variable & Recentering of the running variable

• We will simply subtract 0.5 from the running variable.

```
# Recentering
lmb data <- lmb data %>%
  mutate(demvoteshare c = demvoteshare - 0.5)
lm 1 <- lm robust(score ~ lagdemocrat + demvoteshare c, data = lmb data, se type = "HC1")</pre>
lm_2 <- lm_robust(score ~ democrat + demvoteshare_c, data = lmb_data, se_type = "HC1")</pre>
lm_3 <- lm_robust(democrat ~ lagdemocrat + demvoteshare_c, data = lmb_data, se_type = "HC1")</pre>
screenreg(l = list(lm_1, lm_2, lm_3),
          digits = 2,
          # caption = 'title',
          custom.model.names = c("ADA t+1", "ADA t", "DEM t+1"),
          include.ci = F.
          include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
          include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
          custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
          # select coefficients to report
          stars = numeric(0))
```

##				
##	=============	============	=============	==========
##		ADA_t+1	ADA_t	DEM_t+1
##				
##	lagdemocrat	33.45		0.55
##		(0.85)		(0.01)
##	democrat		58.50	
##			(0.66)	
##				
##	Num. obs.	13577	13577	13577
##	============	============	=============	=========

#### Different slopes on either side of the discontinuity

 How to impliment a regression line to be on either side, which means necessarily that we have two lines left and right of the discontinuity? ⇒ Interaction

```
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c,</pre>
                  data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c,</pre>
                  data = lmb data, se type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat*demvoteshare_c,</pre>
                  data = lmb data, se type = "HC1")
screenreg(l = list(lm_1, lm_2, lm_3),
          digits = 2,
          # caption = 'title',
          custom.model.names = c("ADA t+1", "ADA t", "DEM t+1"),
          include.ci = F,
          include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
          include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
          custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
          # select coefficients to report
          stars = numeric(0)
```

##				
##	=============	============	=============	==========
##		ADA_t+1	ADA_t	DEM_t+1
##				
##	lagdemocrat	30.51		0.53
##		(0.82)		(0.01)
##	democrat		55.43	
##			(0.64)	
##				
##	Num. obs.	13577	13577	13577
##	============	============	=============	=========

### Different quadratic regressions in all data

```
lmb data <- lmb data %>%
  mutate(demvoteshare sg = demvoteshare c^{2})
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,</pre>
                  data = lmb data, se type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c + democrat*demvoteshare_sq,</pre>
                  data = lmb data, se type = "HC1")
lm 3 <- lm robust(democrat ~ lagdemocrat*demvoteshare c + lagdemocrat*demvoteshare sq,</pre>
                  data = lmb data, se type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
          digits = 2,
          # caption = 'title',
          custom.model.names = c("ADA t+1", "ADA t", "DEM t+1"),
          include.ci = F,
          include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
          include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
          custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
          # select coefficients to report
          stars = numeric(0))
```

##				
##	============	===========	============	==========
##		ADA_t+1	ADA_t	DEM_t+1
##				
##	lagdemocrat	13.03		0.32
##		(1.27)		(0.02)
##	democrat		44.40	
##			(0.91)	
##				
##	Num. obs.	13577	13577	13577
##	============	===========	============	=========

• The larger standard error due to the longer polynomial term.

### Different quadratic regression; limited to +/- 0.05

```
lmb subset <- lmb data %>%
  filter(demvoteshare > .45 & demvoteshare < .55) %>%
  mutate(demvoteshare sg = demvoteshare c^2)
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,</pre>
                  data = lmb subset, se type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c + democrat*demvoteshare_sq,</pre>
                  data = lmb subset, se type = "HC1")
lm 3 <- lm robust(democrat ~ lagdemocrat*demvoteshare c + lagdemocrat*demvoteshare sq,</pre>
                  data = lmb subset, se type = "HC1")
screenreg(l = list(lm_1, lm_2, lm_3),
          digits = 2.
          # caption = 'title',
          custom.model.names = c("ADA t+1", "ADA t", "DEM t+1"),
          include.ci = F,
          include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
          include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
          custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
          # select coefficients to report
          stars = numeric(0))
```

## ##				
##		ADA_t+1	ADA_t	DEM_t+1
## ##	lagdemocrat	 7.35		0.17
## ##	dowoolaat	(1.59)	45 10	(0.02)
# # # #	democrat		45.19 (2.68)	
##				
## ##	Num. obs.	2387 ========	2387	2387

### Optimal bandwidth by rdrobust

- The method of optimal bandwidth selection (Calonico, Cattaneo, and Titiunik 2014) can be implemented with the user-created rdrobust command.
- These methods ultimately choose optimal bandwidths that may differ left and right of the cutoff based on some bias-variance trade-off.

## [1] "Mass points detected in the running variable."

## ##	Call: rdrobust					
##	Number of Obs.		13577			
##	BW type		mserd			
##	Kernel		Triangular			
# # # #	VCE method		NN			
##	Number of Obs.		5480	8097		
##	Eff. Number of Ob	os.	2112	1893		
##	Order est. (p)		1	1		
##	Order bias (q)		2	2		
##	BW est. (h)		0.086	0.086		
##	BW bias (b)		0.141	0.141		
##	rho (h/b)		0.609	0.609		
##	Unique Obs.		2770	3351		
##						
##	=======================================	=======		=================	========	
##	Method	Coef.	Std. Err.	Z	P> z	[ 95% C.I. ]
##	=======================================	========	=================	=================	========	
##		46.491	1.241			- , -
##	Robust	-	-	31.425	0.000	[43.293 , 49.052]
##	=======================================	=========	================	================	=========	

# **Covariate test**

### Covariates by the running variables

- We use income (realincome) as covariates.
- We limit window of voteshare from 0.25 to 0.75.

```
#aggregating the data
lmb_subset = lmb_data %>%
    dplyr::select(realincome,demvoteshare) %>%
    filter(demvoteshare>.25 & demvoteshare<.75) %>%
    na.omit()
#calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_subset$realincome, cut(lmb_subset$demvoteshare, 50)) %>%
    lapply(mean) %>%
    unlist()
```

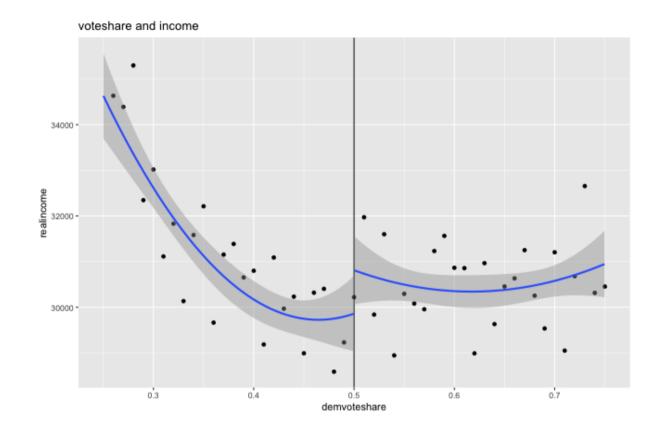
```
#createing new data frame for plotting
agg_lmb_data <- data.frame(income = demmeans, demvoteshare = seq(0.26, 0.75, by = 0.01))</pre>
```

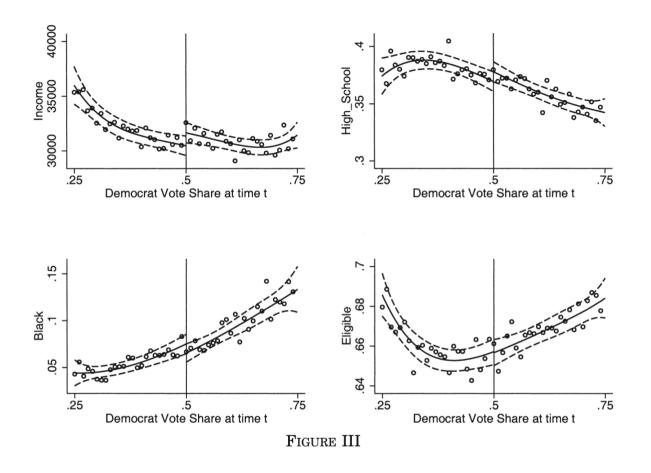
#### **Covariate test for income**

```
#grouping above or below threshold
lmb_subset <- lmb_subset %>%
   mutate(gg_group = if_else(demvoteshare > 0.5, 1,0))
```

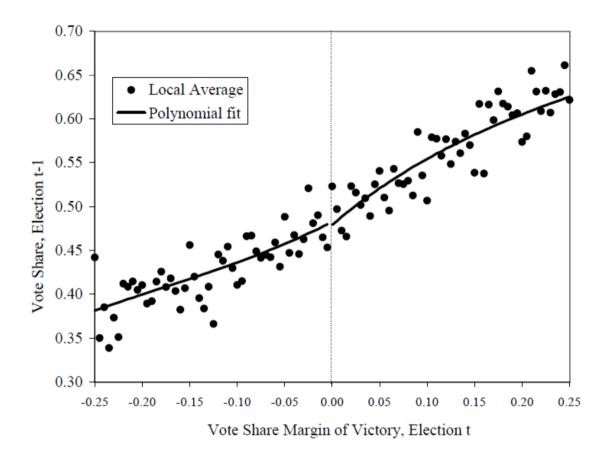
```
#plotting
```

```
ggplot(data=lmb_subset, aes(demvoteshare, realincome)) +
    geom_point(aes(x = demvoteshare, y = income), data = agg_lmb_data) +
    geom_vline(xintercept = 0.5) +
    stat_smooth( aes(demvoteshare, realincome, group = gg_group), method = "lm", formula = y ~ x + I()
```





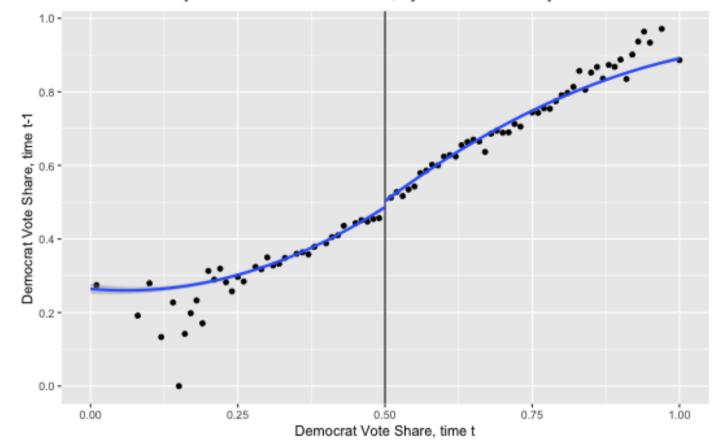
• The authors also did covariate tests with other variables such as percentage with highschool degree (pcthighschl), percentage black (pctblack), percentage eligible to vote (votingpop/totpop).



• t-1 period's outcome is also often used as a placebo.

#### Coding of placebo

```
#aggregating the data
# calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_data$lagdemvoteshare, cut(lmb_data$demvoteshare, 100)) %>%
     lapply(mean) %>%
     unlist()
#createing new data frame for plotting
agg lmb data <- data.frame(lagdemvoteshare=demmeans, demvoteshare = seg(0.01,1, by = 0.01))
#grouping above or below threshold
lmb data <- lmb data %>%
     mutate(gg_group = if_else(demvoteshare > 0.5, 1,0))
#plotting
ggplot(data=lmb data, aes(demvoteshare, lagdemvoteshare)) +
             geom_point(aes(x = demvoteshare, y = lagdemvoteshare), data = agg_lmb_data) +
            xlim(0,1) + ylim(-0.1,1.1) +
            geom vline(xintercept = 0.5) +
            xlab("Democrat Vote Share, time t") +
            ylab("Democrat Vote Share, time t-1") +
             scale y continuous(breaks=seg(0,1,0.2)) +
             ggtitle(TeX("Democratic Party Vote Share in Election t-1, by Democratic Party 
                                                                   method = "lm", formula = v \sim x + I(x^2))
```



Democratic Party Vote Share in Election t-1, by Democratic Party Vote Share in Electic

# **Density test**

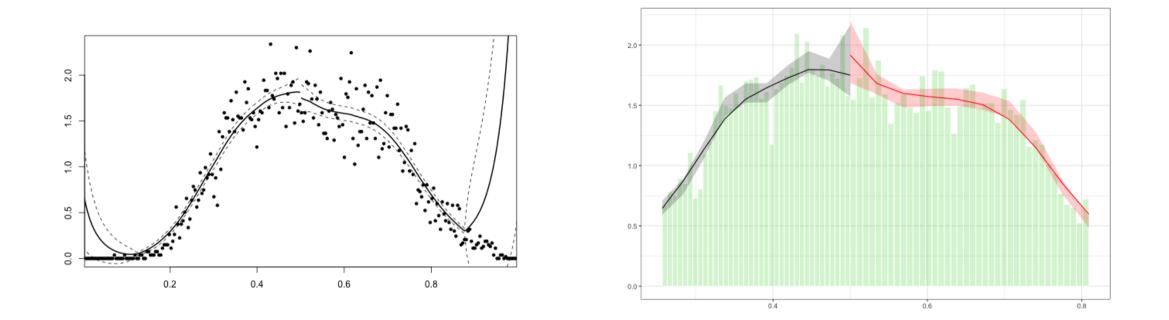
## Density of the running variables

- McCrary density test
- We will implement this test using local polynomial density estimation (Cattaneo, Jansson, and Ma 2019).

```
# install.packages("rddensity")
# install.packages("rdd")
library(rddensity)
library(rdd)
```

```
DCdensity(lmb_data$demvoteshare, cutpoint = 0.5)
```

```
density <- rddensity(lmb_data$demvoteshare, c = 0.5)
rdplotdensity(density, lmb_data$demvoteshare)</pre>
```



• No signs that there was manipulation in the running variable at the cutoff.