# Regression Discontinuity 2: Implementation in R 

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## Close election design

## Question and Estimation strategy

- Lee, D.S., Moretti, E., and M. Butler, 2004, Do Voters Affect or Elect Policies? Evidence from the U.S. House, Quarterly Journal of Economics 119, 807-859.
- Do voters affect policy itself or do they just select politician?
- The roll-call voting record $R C_{t}$ of the representative in the district following the election t can be written as

$$
R C_{t}=\left(1-D_{t}\right) y_{t}+D_{t} x_{t}
$$

- $D_{t}$ : indicator variable for whether the Democrat won election t
- $x_{t}\left(y_{t}\right)$ : the policy implemented by the Democrat (the Repulican) at t
- Under some conditions, it can be expressed as

$$
\begin{align*}
R C_{t} & =\text { constant }+\pi_{0} P_{t}^{*}+\pi_{1} D_{t}+\epsilon_{t} \\
R C_{t+1} & =\text { constant }+\pi_{0} P_{t+1}^{*}+\pi_{1} D_{t+1}+\epsilon_{t+1} \tag{2}
\end{align*}
$$

- $P_{t}^{*}$ : voters' underlying popularity (the electoral strength) of the Democrat. It is defined as the probability that party D will win if parties D and R are expected to choose their blis points, not moderating points.
- What we try to know is whether $\pi_{0}=0$ or $\pi_{1}=0$, or neither, meaning what affect representative's roll-call voting, in other words, politician's decision.
- If $\pi_{1}=0$, the roll-call voting of the representative in the district does not vary regarless of who wins (called Complete Convergence). That is both parties choose the exactly same policy. The policy position is determined only by the voter's underlying popularity.
- If $\pi_{0}=0$, the roll-call voting of the representative in the district does not affected by voters' underlying popularity (called Complete Divergence). This can be interpretted that voters can not affect policy, but merely elect politicians' fixed policies.
- If else, both parties select different policies, but voters can affect policy (called Partial Convergence).
- The problem is we cannot estimate equations (1) and (2), because we cannot observe $P_{t}^{*}$.
- This brings two issues to figure out in order to identify $\pi_{0}$ and $\pi_{1}$.

1. Simple comparison of $R C_{t}$ between $D_{t}=1$ and $D_{t}=0$ without controlling on $P_{t}^{*}$ leads endogeneity bias, since $P_{t}^{*}$ tends to be higher among $D_{t}=1$.
$\Rightarrow$ We need to somehow control $P_{t}^{*} \Rightarrow$ RDD

- By focusing on close elections (when voteshares of both parties are very tight), we can compare the cases between when $D_{t}=1$ and $D_{t}=0$, fixing $P_{t}^{*}$ constant. $\Rightarrow$ Being able to identify $\pi_{1}$.

2. Because $P_{t}^{*}$ is directly unobservable, we have to somehow find variation of $P_{t}^{*}$ to identify $\pi_{0}$.
```
| Incumbency advantage
```

- The random assignment of who wins in the first election generates random assignment in which candidate has greater electoral strength for the next election. $\Rightarrow$ This requires two period analysis.


## Identification

- The conditional expectation of equation (2) is:

$$
\begin{array}{r}
\lim _{v \downarrow 0.5} E\left[R C_{t+1} \mid V_{t}=v\right]=\text { constant }+\pi_{0} E\left[P_{t+1}^{*} \mid D_{t}=1, V_{t}=0.5\right] \\
+\pi_{1} E\left[D_{t+1} \mid D_{t}=1, V_{t}=0.5\right] \\
=\text { constant }+\pi_{0} P_{t+1}^{* D}+\pi_{1} P_{t+1}^{D}
\end{array}
$$

$$
\begin{aligned}
& \lim _{v \uparrow 0.5} E\left[R C_{t+1} \mid V_{t}=v\right]=\text { constant }+\pi_{0} E\left[P_{t+1}^{*} \mid D_{t}=0, V_{t}=0.5\right] \\
&+\pi_{1} E\left[D_{t+1} \mid D_{t}=0, V_{t}=0.5\right] \\
&= \text { constant }+\pi_{0} P_{t+1}^{* R}+\pi_{1} P_{t+1}^{R}
\end{aligned}
$$

- $V_{t}$ is voteshare of the Democrat in election t , and threshold is 0.5 .
- $P_{t+1}^{* D} \equiv E\left[P_{t+1}^{*} \mid D_{t}=1, V_{t}=0.5\right], P_{t+1}^{* R} \equiv E\left[P_{t+1}^{*} \mid D_{t}=0, V_{t}=0.5\right]$
- $P_{t+1}^{D}\left(P_{t+1}^{R}\right)$ is equilibrium probability that Democrat wins in election $t+1$ when Democrat (Republican) won in election t .


## Estimation

- When one could randomize $D_{t}$ by restricting data close to the threshold,

$$
\begin{align*}
\underbrace{E\left[R C_{t+1} \mid D_{t}=1\right]-E\left[R C_{t+1} \mid D_{t}=0\right]}_{\text {Observable }} & =\pi_{0}\left(P_{t+1}^{* D}-P_{t+1}^{* R}\right)+\pi_{1}\left(P_{t+1}^{D}-P_{t+1}^{R}\right) \\
& \equiv \underbrace{\gamma}_{\text {Total effect of initial win on future roll call votes }}
\end{align*}
$$

$$
\begin{align*}
& \underbrace{E\left[R C_{t} \mid D_{t}=1\right]-E\left[R C_{t} \mid D_{t}=0\right]}_{\text {Observable }}=\pi_{1}  \tag{4}\\
& \underbrace{E\left[D_{t+1} \mid D_{t}=1\right]-E\left[D_{t+1} \mid D_{t}=0\right]}_{\text {Observable }}=P_{t+1}^{D}-P_{t+1}^{R}
\end{align*}
$$

- Therefore, $\pi_{0}\left(P_{t+1}^{* D}-P_{t+1}^{* R}\right)$ can be estimated by $\gamma-\pi_{1}\left(P_{t+1}^{D}-P_{t+1}^{R}\right)$


## Data

- There are two main data sets in this project.
- The first is a measure of how liberal an official voted, broght from ADA score for 1946-1995. ADA varies from 0 to 100 for each representative. Higher scores correspond to a more "liberal" voting record.
- The running variable in this study is the vote share. That is the share of all votes that went to a Democrat across Congressional districts.
- U. S. House elections are held every two years.
- Panel data (1946-1995 $\times$ all districs around the U.S.)
- Main variables
- score: ADA score in Congressional session t of the representative elected at $\mathrm{k}\left(R C_{t}\right)$
- democrat: indicator whether the Democrat wins in election $t\left(D_{t}\right)$
- lagdemocrat: indicator whether the Democrat wins in election t-1 $\left(D_{t-1}\right)$
- demvoteshare: voteshare at district k in election $\mathrm{t}\left(V_{t}\right)$
- lagdemvoteshare: voteshare at district k in the previous election, $\mathrm{t}-1\left(V_{t-1}\right)$
- For example, one specific row of the dataset has the voteshares and the results of the November 1992 election (period t) and the November 1990 election (period t -1) at district k , and the ADA score of 1993-1994 Congressional session (period t ).


## Graphical Analysis

## Graphical Analysis

- Results of the analysis will be seen later. Here, we learn how to implement graphical analysis first.
- RD analyses hinge on their graphical analyses.
- always start with visual inspection to check which model (e.g. linear or nonlinear) is plausible.


## Outcomes by the running variables

- First, we try to create this figure from the article.

- The dependent variable is probability of Democrat victory in election $t+1$ and the independent is voteshare in election $t$.
- Then, we will see what happens when we change bandwidth and functional form.

```
library(tidyverse)
library(haven)
library(estimatr)
library(texreg)
library(latex2exp)
# Download data
read_data <- function(df)
{
    full_path <- paste("https://raw.github.com/scunning1975/mixtape/master/",
                                    df, sep = "")
    df <- read_dta(full_path)
    return(df)
}
lmb_data <- read_data("lmb-data.dta")
```

- First, you have more than 10,000 data points, so reduce them for scatter plot.

```
#aggregating the data
# calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_data$democrat, cut(lmb_data$lagdemvoteshare, 100)) %>%
    lapply(mean) %>%
    unlist()
#createing new data frame for plotting
agg_lmb_data <- data.frame(democrat = demmeans, lagdemvoteshare = seq(0.01,1, by = 0.01))
```


## Quadratic fitting in all data

```
#grouping above or below threshold
lmb_data <- lmb_data %>%
    mutate(gg_group = if_else(lagdemvoteshare > 0.5, 1,0))
#plotting
gg_srd = ggplot(data=lmb_data, aes(lagdemvoteshare, democrat)) +
    geom_point(aes(x = lagdemvoteshare, y = democrat), data = agg_lmb_data) +
    xlim(0,1) + ylim(-0.1,1.1) +
    geom_vline(xintercept = 0.5) +
    xlab("Democrat Vote Share, time t") +
    ylab("Probability of Democrat Win, time t+1") +
    scale_y_continuous(breaks=seq(0,1,0.2)) +
    ggtitle(TeX("Effect of Initial Win on Winning Next Election: $\\P^D_{t+1} - P^R_{t+1}$"))
gg_srd + stat_smooth(aes(lagdemvoteshare, democrat, group = gg_group),
            method = "lm", formula = y ~ x + I (x^2))
```

Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


## Quadratic fitting; limited to +/- 0.05

```
gg_srd + stat_smooth(data=lmb_data %>% filter(lagdemvoteshare>.45 & lagdemvoteshare<.55),
aes(lagdemvoteshare, democrat, group = gg_group),
    method = "lm", formula = y ~ x + I(x^2))
```

Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


- Notice that confidence interval widens. But, lines fit plots better.


## Linear different slops

```
gg_srd + stat_smooth(aes(lagdemvoteshare, democrat, group = gg_group), method = "lm")
```

Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


## Linear common slop

```
gg_srd + stat_smooth(data=lmb_data, aes(lagdemvoteshare, democrat),
    method = "lm", formula = y ~ x + I(x > 0.5))
```

- Alternatively, this can avoid showing line across the threshold.

```
lm_tmp <- lm(democrat ~ lagdemvoteshare + I(lagdemvoteshare>0.5), data = lmb_data)
lm_fun <- function(x) predict(lm_tmp, data.frame(lagdemvoteshare = x)) #output is predicted democra
gg_srd +
stat_function(
    data = data.frame(x = c(0, 1),y = c(0, 1)),aes(x = x,y=y),
    fun = lm_fun,xlim = c(0,0.499),
    col="blue",size = 1.5) +
stat_function(
    data = data.frame(x = c(0, 1),y = c(0, 1)),aes(x = x,y=y),
    fun = lm_fun,xlim = c(0.501,1),
    col="blue", size = 1.5 )
```

Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


## Loess fitting

gg_srd + stat_smooth(aes(lagdemvoteshare, democrat, group = gg_group), method = "loess")


- Compared to the quadratic case, variance gets bigger but the prediction fits the points better.


## Kernel-weighted local polynomial regressions

```
library(stats)
smooth_dem0 <- lmb_data %>%
    filter(lagdemvoteshare < 0.5) %>%
    dplyr::select(democrat, lagdemvoteshare) %>%
    na.omit()
smooth_dem0 <- as_tibble(ksmooth(smooth_dem0$lagdemvoteshare, smooth_dem0$democrat,
                            kernel = "box", bandwidth = 0.1))
smooth_dem1 <- lmb_data %>%
    filter(lagdemvoteshare >= 0.5) %>%
    dplyr::select(democrat, lagdemvoteshare) %>%
    na.omit()
smooth_dem1 <- as_tibble(ksmooth(smooth_dem1$lagdemvoteshare, smooth_dem1$democrat,
                            kernel = "box", bandwidth = 0.1))
gg_srd +
    geom_smooth(aes(x, y), data = smooth_dem0) +
    geom_smooth(aes(x, y), data = smooth_dem1)
```

Effect of Initial Win on Winning Next Election: $P_{t+1}^{D}-P_{t+1}^{R}$


## Model and Bandwidth selection - bias-variance tradeoff

- How should we pick the "right" model and bandwidth?
- There's always a trade-off between bias and variance when choosing bandwidth and polynomial length.
- Bias: distance between your prediction and true value
- Variance: width of your prediction
- The shorter the window and the more flexible (e.g. higher-order polynomials) the model, the lower the bias, but because you have less data, the variance in your estimate increases.
- Always, it's important to show robustness.
- Model selection
- Higher-order polynomials can lead to overfitting (Gelman and Imbens 2019). They recommend using local linear regressions with linear and quadratic forms only.
- Local linear regression with a kernel smoother is a popular choice
- Bandwidth selection:
- Optimal bandwidth selection: Imbens and Kalyanaraman (2011), Calonico, Cattaneo, and Titiunik (2014) implimentation will be at the last slide
- Cross validation: Imbens and Lemieux (2008)


## Quantitative analysis

## Quantitative analysis

- Our next goal is to replicate the quantitaive results of Lee, Moretti, and Butler (2004) in the table below.

|  | $\gamma$ | $\pi_{1}$ | $P_{t+1}^{D}-P_{t+1}^{R}$ | $\pi_{1}\left(P_{t+1}^{D}-P_{t+1}^{R}\right)$ | $\pi_{0}\left(P_{t+1}^{* D}-P_{t+1}^{* R}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :--- |
| Variable | $A D A_{t+1}$ | $A D A_{t}$ | $D E M_{t+1}$ |  |  |
| Estimated gap 21.2 (1.9) | $47.6(1.3)$ | $0.48(0.02)$ |  |  |  |

- The analysis restrics only observations where the Democrat voteshare is between 48 percent and 52 percent, so that the number of observations is 915 .
- From the second column, complete convergence is rejected.
- The last column of the statistical insignificance shows that voters primarily elect policies rather than affect policies.
- Complete divergence is supported by this analysis.

```
# Restrict data containg the Democrat vote share between 48 percent and 52 percent
# `lagdemvoteshare` is the Dem. voteshare of the t-1 period
lmb_subset <- lmb_data %>%
    filter(lagdemvoteshare>.48 & lagdemvoteshare<.52)
# E[ADA_{t+1}|D_t] = Igamma
lm_1 <- lm_robust(score ~ lagdemocrat, data = lmb_subset, se_type = "HC1")
# E[ADA_{t}|D_t] = |pi_1
lm_2 <- lm_robust(score ~ democrat, data = lmb_subset, se_type = "HC1")
# E[D_{t+1}/D\mp@subsup{D}{-}{\prime}t]=\mp@subsup{P}{-}{\prime}{t+1}^DD - P_{t+1}^^R
lm_3 <- lm_robust(democrat ~ lagdemocrat, data = lmb_subset, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



- The results are slightly different. But ignore that for now.
- From now on, we will see how the results depend on bandwidth and fanctional form.


## Same specification in all the data

```
#using all data (note data used is lmb_data, not lmb_subset)
lm_1 <- lm_robust(score ~ lagdemocrat, data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat, data = lmb_data, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat, data = lmb_data, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



- Here we see that simply running the regression yields different estimates when we include data far from the cutoff itself.


## Controls for the running variable \& Recentering of the running variable

- We will simply subtract 0.5 from the running variable.

```
# Recentering
lmb_data <- lmb_data %>%
    mutate(demvoteshare_c = demvoteshare - 0.5)
lm_1 <- lm_robust(score ~ lagdemocrat + demvoteshare_c, data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat + demvoteshare_c, data = lmb_data, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat + demvoteshare_c, data = lmb_data, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



## Different slopes on either side of the discontinuity

- How to impliment a regression line to be on either side, which means necessarily that we have two lines left and right of the discontinuity? $\Rightarrow$ Interaction

```
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c,
    data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c,
    data = lmb_data, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat*demvoteshare_c,
    data = lmb_data, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



## Different quadratic regressions in all data

```
lmb_data <- lmb_data %>%
    mutate(demvoteshare_sq = demvoteshare_c^2)
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,
    data = lmb_data, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c + democrat*demvoteshare_sq,
    data = lmb_data, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,
    data = lmb_data, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



- The larger standard error due to the longer polynomial term.


## Different quadratic regression; limited to $+/-0.05$

```
lmb_subset <- lmb_data %>%
    filter(demvoteshare > . 45 & demvoteshare < .55) %>%
    mutate(demvoteshare_sq = demvoteshare_c^2)
lm_1 <- lm_robust(score ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,
    data = lmb_subset, se_type = "HC1")
lm_2 <- lm_robust(score ~ democrat*demvoteshare_c + democrat*demvoteshare_sq,
    data = lmb_subset, se_type = "HC1")
lm_3 <- lm_robust(democrat ~ lagdemocrat*demvoteshare_c + lagdemocrat*demvoteshare_sq,
    data = lmb_subset, se_type = "HC1")
screenreg(l = list(lm_1, lm_2,lm_3),
    digits = 2,
    # caption = 'title',
    custom.model.names = c("ADA_t+1", "ADA_t", "DEM_t+1"),
    include.ci = F,
    include.rsquared = FALSE, include.adjrs = FALSE, include.nobs = T,
    include.pvalues = FALSE, include.df = FALSE, include.rmse = FALSE,
    custom.coef.map = list("lagdemocrat"="lagdemocrat","democrat"="democrat"),
    # select coefficients to report
    stars = numeric(0))
```



## Optimal bandwidth by rdrobust

- The method of optimal bandwidth selection (Calonico, Cattaneo, and Titiunik 2014) can be implemented with the user-created rdrobust command.
- These methods ultimately choose optimal bandwidths that may differ left and right of the cutoff based on some bias-variance trade-off.

```
# install.packages("rdrobust")
library(rdrobust)
rdr <- rdrobust(y = lmb_data$score,
    x = lmb_data$demvoteshare, c = 0.5)
summary(rdr)
```

\#\# [1] "Mass points detected in the running variable."

```
## Call: rdrobust
##
## Number of Obs. 13577
## BW type
## Kernel
mserd
Triangular
## VCE method
NN
##
## Number of Obs. 5480 8097
## Eff. Number of Obs. 2112 1893
## Order est. (p) 1 1
## Order bias (q) 2 2
## BW est. (h)
## BW bias (b)
## rho (h/b)
## Unique Obs. 2770 0.609
##
## ==================================================================================
## Method Coef. Std. Err. z P>|z| [ 95% C.I.]
```



```
## Conventional 46.491 1.241 37.477 0.000 [44.060 , 48.923]
## Robust - - 31.425 0.000 [43.293 , 49.052]
```



## Covariate test

## Covariates by the running variables

- We use income (realincome) as covariates.
- We limit window of voteshare from 0.25 to 0.75 .

```
#aggregating the data
lmb_subset = lmb_data %>%
    dplyr::select(realincome,demvoteshare) %>%
    filter(demvoteshare>. }25\mathrm{ & demvoteshare<.75) %>%
    na.omit()
#calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_subset$realincome, cut(lmb_subset$demvoteshare, 50)) %>%
        lapply(mean) %>%
        unlist()
#createing new data frame for plotting
agg_lmb_data <- data.frame(income = demmeans, demvoteshare = seq(0.26, 0.75, by = 0.01))
```


## Covariate test for income

```
#grouping above or below threshold
lmb_subset <- lmb_subset %>%
    mutate(gg_group = if_else(demvoteshare > 0.5, 1,0))
#plotting
ggplot(data=lmb_subset, aes(demvoteshare, realincome)) +
    geom_point(aes(x = demvoteshare, y = income), data = agg_lmb_data) +
    geom_vline(xintercept = 0.5) +
    stat_smooth( aes(demvoteshare, realincome, group = gg_group), method = "lm", formula = y ~ x + I(;
```




- The authors also did covariate tests with other variables such as percentage with highschool degree (pcthighschl), percentage black (pctblack), percentage eligible to vote (votingpop/totpop).

- t-1 period's outcome is also often used as a placebo.


## Coding of placebo

```
#aggregating the data
# calculate mean value for every 0.01 voteshare
demmeans <- split(lmb_data$lagdemvoteshare, cut(lmb_data$demvoteshare, 100)) %>%
    lapply(mean) %>%
    unlist()
#createing new data frame for plotting
agg_lmb_data <- data.frame(lagdemvoteshare=demmeans, demvoteshare = seq(0.01,1, by = 0.01))
#grouping above or below threshold
lmb_data <- lmb_data %>%
    mutate(gg_group = if_else(demvoteshare > 0.5, 1,0))
#plotting
ggplot(data=lmb_data, aes(demvoteshare, lagdemvoteshare)) +
    geom_point(aes(x = demvoteshare, y = lagdemvoteshare), data = agg_lmb_data) +
    xlim(0,1) + ylim(-0.1,1.1) +
    geom_vline(xintercept = 0.5) +
    xlab("Democrat Vote Share, time t") +
    ylab("Democrat Vote Share, time t-1") +
    scale_y_continuous(breaks=seq(0,1,0.2)) +
    ggtitle(TeX("Democratic Party Vote Share in Election t-1, by Democratic Party Vote Share in Eler
                            method = "lm", formula = y ~ x + I (x^2))
```

Democratic Party Vote Share in Election t-1, by Democratic Party Vote Share in Electic


## Density test

## Density of the running variables

- McCrary density test
- We will implement this test using local polynomial density estimation (Cattaneo, Jansson, and Ma 2019).

```
# install.packages("rddensity")
# install.packages("rdd")
library(rddensity)
library(rdd)
DCdensity(lmb_data$demvoteshare, cutpoint = 0.5)
density <- rddensity(lmb_data$demvoteshare, c = 0.5)
rdplotdensity(density, lmb_data$demvoteshare)
```




- No signs that there was manipulation in the running variable at the cutoff.

