Demand Estimation with Flexible Income Effect: An Application to Pass-through and Merger Analysis*

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Abstract

This paper proposes a semiparametric discrete choice model that incorporates a nonparametric specification for income effects. The model allows for the flexible estimation of demand curvature, which has significant implications for pricing and policy analysis in oligopolistic markets. Our estimation algorithm adopts a method of sieve approximation with shape restrictions in a nested fixed-point algorithm. Applying this framework to the Japanese automobile market, we conduct a pass-through analysis of feebates and merger simulations. Our model predicts a higher pass-through rate and more significant merger effects than parametric demand models, highlighting the importance of flexibly estimating demand curvature.

JEL Classification: L1, L41, L62

Keywords: Discrete choice model, differentiated product, income effect, semiparametric model, aggregate data, sieve approximation, shape restriction, pass-through analysis, merger simulation, automobile

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1 Introduction

Estimation of a consumer demand model is a vital part of empirical research in industrial organization and related fields. Price elasticity and the substitution patterns implied by the demand model are key factors firms must consider when making pricing decisions in oligopolistic markets. Consumer demand is also essential for evaluating the welfare consequences of a firm's strategic behavior and policy changes. Therefore, accurate measurement of consumer demand is critical for various applications, including merger analysis (Nevo, 2000a), pass-through analysis of cost shocks and taxes (Weyl and Fabinger, 2013), and the introduction of new products (Petrin, 2002).

Given the practical importance of demand models, a vast body of literature in empirical industrial organization has proposed econometric methods for estimating consumer demand for differentiated products (Berry and Haile, 2021; Gandhi and Nevo, 2021). The majority of existing frameworks rely on parametric specifications because a fully flexible model for differentiated product demand involves a significant number of parameters.[1] However, this approach could be problematic because parametric specifications often impose strong restrictions on the shape of the demand curve, which can influence the implications of supply and demand analysis.

To address this concern, this paper proposes a semiparametric framework for discrete choice demand that flexibly accommodates income effects. We demonstrate that a flexible specification for the income effect is crucial for accurately estimating the curvature of the demand function. Applying the proposed framework to data from the Japanese automobile industry, we conduct merger simulations and pass-through analysis for a feebate policy (i.e., a subsidy for eco-friendly cars). These simulations highlight the practical value of our demand framework in policy-relevant applications.

Following prior research, such as McFadden (1974), Berry (1994), and Berry et al. (1995) (henceforth referred to as BLP), we employ a random utility framework to model the demand for differentiated products. However, our approach distinguishes itself from prior research by incorporating income effects in a nonparametric manner. Previous studies have often used a quasi-linear specification of random utility without considering income effects, or incorporating them based on

[1] Consider a log-log specification for the demand system with J products. The number of parameters required to estimate the own- and cross-price elasticity matrices is on the order of J^2. Consequently, to alleviate the estimation burden, researchers must impose constraints on the demand system. Further details can be found in Berry (1994).

parametric assumptions. We demonstrate that allowing for greater flexibility in the functional form of the income effect term is critical for accurately estimating demand curvature and price elasticity patterns. Furthermore, our framework permits the estimation of welfare changes in the presence of income effects (McFadden 1999; Dagsvik and Karlström 2005).

Our demand model employs a semiparametric framework that includes both a parametric component for utility derived from product characteristics and a nonparametric function to capture the income effect. To estimate this model, we combine a method of sieve approximation from the semiparametric econometrics literature (Chen, 2007) and the nested fixed point algorithm proposed by BLP. We first approximate the nonparametric function of the income effect using a sieve, that is, a linear combination of known basis functions. We select Bernstein polynomials as the basis function due to their shape-preserving properties (explained in greater detail below). After implementing the sieve approximation, our model closely aligns with the standard parametric framework of BLP. We then use a nested fixed point algorithm to implement sieve GMM estimation.

A novel aspect of our estimation method is the exploitation of a shape restriction on the nonparametric function representing the income effect. Non- and semiparametric estimation methods are associated with the imprecision of estimators as a cost of flexibility. This issue becomes particularly critical in the presence of endogeneity, a major challenge encountered in demand estimation.[2] To mitigate this problem, recent econometrics literature has suggested that researchers should employ shape restrictions in such estimations.[3] As demonstrated in Section 2.1, the income effect term is assumed to be weakly increasing, a presumption based on the utility maximization behavior of consumers. We integrate this monotonicity constraint into our semiparametric estimation approach. Monte Carlo experiments, detailed in Section 4, illustrate that incorporating a shape restriction significantly reduces the variance of the estimated nonparametric function in our demand model.[4]

We apply our semiparametric framework to the Japanese automobile market. Our data include product- and market-level information on sales, prices, and characteristics from 2006 to 2013. Our

[2]In a semiparametric setting, endogeneity results in an ill-posed inverse problem, leading to imprecise estimation of the nonparametric components. For further details, see Horowitz (2014).

[3]Chetverikov and Wilhelm (2017) demonstrate that imposing a shape restriction can improve estimation performance in the context of a nonparametric instrumental variable model.

[4]Prior studies, such as Blundell et al. 2017 and Chetverikov and Wilhelm 2017, have demonstrated that imposing shape restrictions can enhance estimation performance in non- or semiparametric models that are linearly separable in the error term. Our simulation analysis suggests that this insight is also applicable to non-separable models.

estimation results reveal significant nonlinearity in the shape of the income effect. We then use the estimated model to conduct two counterfactual simulations: a pass-through analysis and a merger simulation, where the curvature of demand plays a crucial role. We compare the simulation results obtained from our demand model with those derived from parametric logit models with and without consumer heterogeneity.

In our first counterfactual analysis, we evaluate the effects of the Japanese government's subsidy for eco-friendly cars introduced in 2009. This evaluation involves a pass-through analysis specifically focused on a subsidy. Theoretically, Weyl and Fabinger (2013) demonstrated that the curvature of demand determines the degree of pass-through to final prices. Our semiparametric model predicts much higher pass-through rates compared to a quasi-linear logit model (without consumer heterogeneity), which imposes a priori restrictions on the pass-through rate. Compared to a parametric random coefficient logit model, the semiparametric model shows more significant heterogeneity in the pass-through rates across products.

In our second counterfactual analysis, we conduct a merger simulation using our demand model. The merger simulation is one of the most policy-relevant applications of demand estimation, and thus is suited to demonstrating the value of our semiparametric model. As discussed in Farrell and Shapiro (2010), the price effects of a merger can be considered as, in the first order, the pass-through of the increase in the opportunity costs, known as the upward pricing pressure (UPP). We find that our semiparametric model predicts the largest price effects of a merger among three demand specifications. This result is driven by both the pass-through rate and the UPP implied from our semiparametric model.

The remainder of this paper is organized as follows. First, we review the related literature to clarify the intended contributions of our work. In Section 2, we introduce a demand model for differentiated products incorporating a nonparametric specification for income effect. Section 3 discusses the estimation method using aggregate (market-level) data. In Section 4, we conduct Monte Carlo experiments to evaluate our framework's performance. The framework is subsequently applied to data from the Japanese automobile market, with the demand model estimated in Section 5. Using this estimated model, two counterfactual simulations involving pass-through and merger analyses are presented in Section 6. The robustness of our empirical findings is examined in Section 7. Finally, Section 8 provides the conclusions of our study.

Related Literature This paper contributes to three distinct strands of literature. Firstly, it relates to the literature on non- and semiparametric estimation of consumer demand models. While many previous works (e.g., Blundell et al. 2012, 2017) have focused on the demand for homogenous goods, more recent studies have instead explored the demand for differentiated products within the discrete choice framework (e.g., Bhattacharya 2015; Tebaldi et al. 2019). These studies have primarily investigated nonparametric identification, and estimated demand and welfare based on individual-level data. In contrast, our work addresses scenarios where only aggregate data, such as market-level data, are available. This approach aligns with that taken by BLP and subsequent studies.

Griffith et al. (2018) represents the study most closely related to ours. Their demand model integrates a flexible and parametric form of the income effect within a discrete-choice framework. Using this model, the authors estimated the demand for margarine in the UK and evaluated the impact of a tax on saturated fat content. Our paper complements the work of Griffith et al. (2018) in both methodology and application. Methodologically, we estimate the income effect term nonparametrically with shape restrictions through a sieve approximation, demonstrating that shape restrictions can significantly enhance the precision of estimating the income effect. Furthermore, our analysis focuses on scenarios where only aggregate data (i.e., market-level data) are available, thus contrasting with the individual-level choice data used by Griffith et al. (2018).[5] Additionally, we address price endogeneity through the use of instrumental variables (IVs). Our application investigates automobile demand, highlighting the pivotal role of the income effect in purchasing decisions. We further explore the implications of a flexible income effect in both pass-through analysis and merger simulation.

Other papers related to our work include Compiani (2021), Wang (2022), Birchall et al. (2023), and Miravete et al. (2023). The former two papers considered a non- and semiparametric modelling of consumer demand. Compiani (2021) introduced a fully nonparametric estimation approach for differentiated product demand models. While accommodating a broad range of demand models, Compiani (2021)'s method uncovered challenges concerning its direct application to scenarios with a large number of products.[6] Our framework addresses such scenarios by targeting a one-dimensional

[5] Herriges and Kling (1999) and Morey et al. (2003) also incorporated the nonlinear income effect in a parametric manner to estimate discrete choice models using individual-level data.

[6] Berry and Haile (2014) and Compiani (2021) focused on the identification and estimation of the inverse of mean

nonparametric object that captures the income effect. In comparison, Wang (2022) presented a semiparametric model for differentiated products within a BLP framework, estimating the distribution of random coefficients nonparametrically. Our work diverges by relaxing the functional form assumption on the income effect and estimating it nonparametrically.

The latter two papers considered a parametric modelling for flexible estimation of consumer demand. Birchall et al. (2023), building upon Björnerstedt and Verboven (2016), employed a Box-Cox specification in the discrete choice model to relax the unit demand assumption, thereby enabling a flexible estimation of demand curvature. They applied this framework to estimating the demand for ready-to-eat cereals. Our methodology complements Birchall et al. (2023) by offering a flexible specification for cases of unit demand, suitable for such durable goods as appliances and automobiles. Lastly, a recent paper by Miravete et al. (2023) considered a unit-demand case and adopted a first-order approximation of the Box-Cox transformation for the income effect function. Our model shows greater flexibility by adopting a non-parametric specification for the income effect function. Our framework also explicitly considers the budget constraint and utility maximization, thus allowing for the welfare analysis.

Secondly, our paper contributes to the empirical literature on pass-through effects, as illustrated by such studies as Nakamura and Zerom (2010), Goldberg and Hellerstein (2013), Fabra and Reguant (2014), and Hollenbeck and Uetake (2021). We enrich this body of work by highlighting the significance of flexibly estimating demand curvature in assessing the pass-through of taxes and subsidies via supply-side simulations. Our empirical results resonate with the theoretical insights of Weyl and Fabinger (2013), which underscored the critical role of demand curvature in determining the pass-through rate. Unlike the simple logit model, which is constrained by the limitation that the pass-through rate cannot exceed unity, our demand model accommodates pass-through rates that can exceed one.

Thirdly, our paper enriches the extensive literature on the empirical analysis of horizontal mergers. Since the seminal work of Nevo (2000a), numerous empirical studies have employed simulation analyses in differentiated product markets to assess merger effects on prices and social welfare.[7] Accurate estimation of the demand model is pivotal for precisely forecasting the price

utility, modeled as a function of vectors of prices and market share, resulting in a $2J$ dimensional function, where J represents the number of products.

[7] For example, see Peters (2006) on airline, Fan (2013) on newspaper, Houde (2012) gas station, Gowrisankaran et

effects of a merger, as oligopolistic firms base their pricing strategies on the underlying demand structure. Although the logit model and its variant, the random coefficient logit model, have been frequently used in these analyses, their inherent restrictive curvature properties may inaccurately influence the simulation of merger effects (see, e.g., Crooke et al. 1999). Addressing this limitation, our work illustrates how our demand model's flexible estimation of demand curvature provides a robust alternative for antitrust analyses.

2 Demand Model

2.1 Utility Maximization Problem

This section introduces a model for differentiated product demand that incorporates a nonparametric income effect. We begin with a utility maximization problem that includes both continuous and discrete choices, following McFadden (1981). Consumers face a discrete choice among differentiated goods and a continuous choice concerning the consumption of all others. As we later assume, this continuous choice problem can be conceptualized as the consumption of a numeraire.

Let $U(\mathbf{m}, j)$ denote the direct utility function, where $\mathbf{m}$ is a d_m-dimensional vector representing the consumption of continuous choice goods. The index $j \in \mathbf{J} \equiv \{0, 1, \ldots, J\}$ corresponds to an alternative in the discrete choice decision, with J products available in the market. Specifically, the index $j = 0$ indicates that the consumer opts not to purchase any of the discrete choice goods, a choice referred to as the outside goods.

The utility maximization problem is given by the following:

$$\begin{aligned} \max_{(\mathbf{m},j)\in R_+^{d_m}\times \mathbf{J}} \quad & U(\mathbf{m}, j) \\ \text{s.t.} \quad & \mathbf{P_m}'\mathbf{m} + p_j \leq y_i, \end{aligned} \tag{2.1}$$

where $\mathbf{P_m}$ is a d_m dimensional vector of prices of continuous choice goods, p_j is the price of alternative j, and y_i is income.

Conditional on choice j in the discrete choice, we define the conditional indirect utility function

al. (2015) hospital, Miller and Weinberg (2017) beer, Ohashi and Toyama (2017) automobile, and Björnerstedt and Verboven (2016) on pharmaceutical mergers.

as follows:

$$V(\mathbf{P_m}, y - p_j, j) \equiv \max_{\mathbf{m}\in R_+^{d_m}} U(\mathbf{m}, j) \text{ s.t. } \mathbf{P_m}'\mathbf{m} \leq y_i - p_j. \tag{2.2}$$

Note that we define $p_0 = 0$ as choosing the outside good incurs no costs.

The right-hand side of Equation (2.2) represents a standard utility maximization problem. Hence, the conditional indirect utility function $V(\mathbf{P_m}, y - p_j, j)$ exhibits the following standard properties: (1) homogeneous of degree 0 regarding both $\mathbf{P_m}$ and $(y - p_j)$, (2) increasing in $(y - p_j)$, (3) non-decreasing in $\mathbf{P_m}$, and (4) quasi-convex in both $(y_i - p_j)$ and $\mathbf{P_m}$. We employ some of these properties for deriving shape restrictions for estimation.

To derive a specification of the conditional indirect utility function suitable for estimation, we introduce the following assumption about the direct utility function:

$$U(\mathbf{m}, j) = v(j) + u(\mathbf{m}). \tag{2.3}$$

This assumption suggests that the utility derived from differentiated goods is independent from that of all other goods. Although this may initially appear as restrictive, most discrete-choice demand models implicitly rely on this assumption.[8]

The conditional indirect utility function can now be expressed as:

$$V(\mathbf{P_m}, y - p_j, j) = v(j) + \tilde{V}(\mathbf{P_m}, y - p_j). \tag{2.4}$$

The function $\tilde{V}(\mathbf{P_m}, y - p_j)$ satisfies the four properties implied by the utility maximization problem. In practice, however, the prices of all other goods, denoted as $\mathbf{P_m}$, are not always observable. To deal with this issue, we assume that the continuous good is a numeraire, with its price represented by P^m. This price is thus treated as a price index. Consequently, we obtain $\tilde{V}(P^m, y - p_j) = u\left(\frac{y-p_j}{P^m}\right)$, implying that the utility from a numeraire depends on the disposal income $y - p_j$ after choosing alternative j. Hereafter, both income y and the price of discrete choice goods p_j are deflated by the price index P^m.

[8] This approach excludes the discrete-continuous choice model described by Dubin and McFadden (1984), where the choice of an appliance (i.e., a discrete decision) influences the utility derived from electricity consumption (i.e., a continuous decision). Newey (2007) explored the nonparametric identification of a discrete-continuous choice model when individual choice data are available. Extending our framework to accommodate such scenarios presents a promising avenue for future research.

We now define the income effect term by $f(y - p_j) \equiv \tilde{V}(P^m, y - p_j)$, albeit with a slight abuse of notation. The function $f(y - p_j)$ plays a pivotal role in our empirical framework. Importantly, the income effect term $f(y - p_j)$ is a weakly-increasing function, imposed in the estimation.

2.2 Conditional Indirect Utility Function

Turning to the utility generated by consuming a discrete choice good $v(j)$ in Equation (2.3), we follow the standard specification found in the literature. We introduce an index i, representing a consumer, and denote the utility from a discrete choice good j as v_{ij}.

$$v_{ij} = \beta' X_j + \xi_j + \epsilon_{ij} \text{ for } j = 1, \ldots, J \tag{2.5}$$

$$v_{i0} = \epsilon_{i0} \tag{2.6}$$

X_j is a vector of observable characteristics of product j, and ξ_j represents its unobservable characteristics. ϵ_{ij} is an independent and identically distributed (IID) idiosyncratic shock that follows the type I extreme-value distribution.

We now present the conditional indirect utility function of consumer i when choosing j:

$$V_{ij} = \begin{cases} f(y_i - p_j) + \beta' X_j + \xi_j + \epsilon_{ij} & \text{for } j = 1, \cdots, J \\ f(y_i) + \epsilon_{i0} & \text{for } j = 0 \end{cases}. \tag{2.7}$$

The specification for the indirect utility function follows standard conventions, with the exception of the income-effect term $f(y-p)$. While we refrain from imposing any parametric functional form on this function, we assume that it is a weakly increasing function, as implied by utility maximization in Equation (2.2).

Our specifications can now be compared with those in previous studies. A major and commonly-seen specification is the quasi-linear form, exemplified by $V_{ij} = \alpha(y_i - p_j) + \beta' X_j + \xi_j + \epsilon_{ij}$. In this model, the demand function is independent of the income level y_i because the income term is canceled out when comparing two alternatives.[9] BLP introduces $V_{ij} = \alpha \ln(y_i - p_j) + \beta' X_j + \xi_j + \epsilon_{ij}$,

[9] Another prevalent specification is $V_{ij} = \alpha_i(y_i - p_j) + \beta' X_j + \xi_j + \epsilon_{ij}, \alpha_i = g(z_i)$, where z_i represents consumer i's characteristics, such as income, age, and household size. See Nevo (2001).

incorporating a parametric form for the income effect.[10] Unlike these models, our approach refrains from imposing any parametric assumption on $f(\cdot)$, other than it being an increasing function. We discuss the benefits of such a flexible specification in Section 2.4, exploring its implications across various contexts in the industrial organization literature and applied microeconomics.

2.3 Individual Choice Probability and Market Share

This subsection considers the discrete choice decision based on the conditional indirect utility function obtained above. We then derive the market share equation, which provides the basis for later estimation. Hereafter, we add the index t denoting the market, which is defined by geography, time, or both. The conditional indirect utility function is now denoted by V_{ijt}.

When considering the discrete choice decision, we need to incorporate the budget constraint in the original utility maximization problem (2.1). The budget constraint implies that consumer i with income y_{it} cannot buy goods whose price p_{jt} is higher than their income. Prior studies have mostly ignored this budget constraint, possibly due to its having no impact under the common utility specification, such as a quasi-linear form. Recent papers (e.g., Xiao et al. 2017; Pesendorfer et al. 2023) have discussed the bias associated with the omitted variable constraint. The budget constraint implies that each consumer i has a different choice set according to their income. Specifically, we define the choice set of consumers i as:

$$\mathbf{J}_{it} = \{0\} \cup \{j \in \{1, \cdots, J_t\} : y_{it} - p_{jt} \geq 0\}, \tag{2.8}$$

where J_t is the total number of products available in market t.

Given the conditional indirect utility V_{ijt}, a consumer i chooses the alternative that provides the highest utility from the choice set $\mathbf{J}_{it}$. The discrete choice problem is given as follows:

$$\max_{j \in \mathbf{J}_{it}} V_{ijt}. \tag{2.9}$$

[10] Practically, the term $\alpha \log(y_i - p_j)$ can be approximated by $\frac{\alpha}{y_i} p_j$ as a first-order Taylor expansion, according to Berry et al. (1999).

The choice probability for consumer i selecting alternative j is derived as

$$s_{ijt}(y_{it}) = \frac{\mathbf{1}\{y_{it} \geq p_{jt}\} \cdot \exp\left(f(y_{it} - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)}{\exp(f(y_{it})) + \sum_{k=1}^{J_t} \mathbf{1}\{y_{it} \geq p_{kt}\} \cdot \exp\left(f(y_{it} - p_{kt}) + \beta' X_{kt} + \xi_{jt}\right)}. \tag{2.10}$$

Note that the indicator function $\mathbf{1}\{y_{it} \geq p_{jt}\}$ accounts for the budget constraint.

The market share of each product s_{jt} is now derived by aggregating the individual choice probability across consumers. In the current specification, consumer heterogeneity comes from individual income y_{it}.[11] Let y_{it} follow the distribution of income $G_t(y_{it})$. The market share is given by

$$s_{jt} = \int s_{ijt}(y_{it}) dG_t(y_{it}).$$

Market demand q_{jt} is calculated by multiplying the market share s_{jt} by the market size N_t, that is, $q_{jt} = N_t \times s_{jt}$.

2.4 Model Implications

This subsection addresses the practical significance of the flexible income effect. We first illustrate its impact on price elasticity, before examining how the shape of the demand function is related to the pass-through analysis and merger simulations.

Price Elasticity To underscore the innovation of our demand specification, we begin by examining the price elasticity in a multinomial logit model characterized by quasi-linear utility without consumer heterogeneity (i.e., $f(y-p) = \alpha(y-p)$ where $\alpha > 0$ is a scalar parameter). Omitting index t for notational simplicity, the own- and cross-price elasticity η_{jk} is given as follows:

$$\eta_{jk} = \frac{\partial q_j}{\partial p_k} \frac{p_k}{q_j} = \begin{cases} -\alpha p_j (1 - s_j) & \text{if } k = j \\ \alpha p_k s_k & \text{if } k \neq j \end{cases} \tag{2.11}$$

Price elasticity under the quasi-linear specification has been shown to be relatively restrictive, as highlighted by Nevo (2000b). More specifically, the absolute value of the own-price elasticity, $|\eta_{jj}|$,

[11] The model can also incorporate random coefficients on product characteristics, e.g., $u_{ijt} = f(y_{it} - p_{jt}) + \beta_i' X_{jt} + \xi_{jt} + \epsilon_{ijt}$, where $\beta_i \sim N(\bar{\beta}, \Sigma)$. This specification allows for a richer substitution pattern across products. See Section 3.2 for details.

grows with its own price p_j, implying that more luxurious goods exhibit higher own-price elasticity.

In our demand specification, the price elasticity is given as follows:

$$\eta_{jk} = \frac{\partial q_j}{\partial p_k}\frac{p_k}{q_j} = \begin{cases} -\frac{p_j}{s_j}\int f'(y_i - p_j)s_{ij}(1 - s_{ij})dG(y_i) & \text{if } k = j \\ \frac{p_k}{s_j}\int f'(y_i - p_k)s_{ij}s_{ik}dG(y_i) & \text{if } k \neq j \end{cases} \tag{2.12}$$

Price elasticity is now influenced by the income effect term $f(y-p)$. More precisely, the curvature of the demand curve, as indicated by its second-order derivative, is determined by the shape of $f(y-p)$, which can be flexibly estimated using a nonparametric approach. Consequently, our methodology avoids imposing any predetermined restrictions on how own-price elasticity varies with price.

It is worth discussing how our approach relates to the random coefficient logit model, which is commonly used in practice. Nevo (2000b) observed that, by introducing heterogeneity in consumers' price sensitivity (i.e., marginal utility from income), it is possible to flexibly estimate price elasticity, even when prices are linearly incorporated into the utility function. This means that, if consumers purchasing less expensive products are highly sensitive to price changes, the own-price elasticity for these products could be significant. Therefore, incorporating a random coefficient can effectively address common issues related to both cross- and own-price elasticity. In our demand model, price sensitivity, denoted by $f'(y_i - p_j)$, varies among consumers based on their income levels. Moreover, in our model, price sensitivity $f'(y_i - p_j)$ also varies with the price level itself, providing additional flexibility in estimating own-price elasticity.[12]

Pass-Through Analysis The pass-through analysis evaluates the extent to which prices adjust in response to changes in production costs or taxes (subsidies). The role of demand curvature as a determinant of the pass-through rate has been emphasized in some theoretical studies. Most notably, Weyl and Fabinger (2013) demonstrated that, in symmetric oligopoly or monopoly markets, the pass-through rate is less than one if and only if the demand curve is log-concave, that is $\frac{d^2 \log q(p)}{dp^2} < 0$. The demand curve becomes log-concave in a multinomial logit model with a quasi-

[12] While our discussion primarily focuses on own-price elasticity, it is worth noting that the income effect term $f(y-p)$ also influences cross-price elasticity. This term introduces consumer heterogeneity in terms of price sensitivity into the demand model, where price-sensitive customers are more inclined to purchase cheaper products. Such heterogeneity fosters distinct substitution patterns according to the product's price.

linear specification.

However, even without adopting a quasi-linear specification, imposing a functional form on the income effect can inherently restrict the demand curvature and, consequently, the pass-through rate. This limitation underscores the significance of our flexible demand framework for analyzing the impact of cost shocks and tax policy on pass-through rates.[13] In our empirical application in Section 5 and 6, we illustrate the significance of the flexible income effect by analyzing the pass-through of a subsidy on eco-friendly cars.

Merger Analysis Merger analysis holds critical importance for antitrust practice, primarily aiming to predict the potential price increases resulting from the merger of two competing firms. One method involves calculating UPP, determined by the diversion ratio of competing products and their markup, as discussed by Farrell and Shapiro (2010) and Jaffe and Weyl (2013). This calculation provides an indication of the merger's possible impact on prices. A more detailed method is merger simulation, where a demand and supply model of the market is constructed and estimated. This approach involves performing a counterfactual simulation to determine the market equilibrium when the merging firms jointly maximize profits.

The shape of the demand function is crucial for estimating the merger effect in both approaches. In the approach proposed by Farrell and Shapiro (2010) and Jaffe and Weyl (2013), the impact of a merger is essentially viewed as a form of cost pass-through in the first order. This is because the merged entity considers the opportunity cost associated with the profit loss from its partner.[14] The price effect of the merger hinges on the pass-through of this increased opportunity cost into the final price. Therefore, the relevance of demand curvature in pass-through analysis is equally applicable here. Furthermore, in merger simulations, Crooke et al. (1999) showed that consumer demand with identical elasticities, but differing curvatures, can result in vastly different simulated merger outcomes.

[13] Griffith et al. (2018) offered in-depth discussions on the critical role of flexible income effect specifications in pass-through analysis.

[14] Consider a merger between two single-product firms. After the merger, the new first-order condition for Firm 1's pricing is $p_1 + \left(\frac{\partial q_1}{\partial p_1}\right)^{-1} q_1 = c_1 + \left(-\frac{\partial q_2}{\partial q_1}\right)^{-1} (p_2 - c_2)$. Here, the left-hand side represents the marginal revenue from Firm 1's product, while the second term on the right-hand side accounts for the profit loss of Firm 2's product due to diversion from Firm 2 to 1, regarded as the opportunity cost for Firm 1 to increase its output. See Section 6.3 for details.

3 Estimation Method

This section presents the estimation method for our semiparametric model, which includes the nonparametric function $f(y-p)$ and the linear parameter β in the utility function. To estimate these model components, we employ a sieve approximation for the nonparametric function and incorporate it into the nested fixed-point (NFP) algorithm, as proposed by BLP. Additionally, we apply a shape restriction to the nonparametric component with the aim of enhancing the precision of the parameter estimation.

3.1 Sieve Approximation with Shape Restriction

We first introduce the sieve approximation method as proposed by Chen (2007) and Blundell et al. (2007). This method involves approximating a nonparametric function through a linear combination of known basis functions. Among various candidates for these functions, we opt for the Bernstein polynomial due to its convenience in incorporating shape restrictions. More specifically, we approximate the function $f(\cdot)$ by the K-th order Bernstein polynomial $B_K(x)$:

$$f(x) \approx B_K(x) = \sum_{k=0}^{K} \pi_k b_k^K(x) \equiv \psi^K(x)'\Pi \tag{3.1}$$

where

$$b_k^K(x) = \begin{pmatrix} K \\ k \end{pmatrix} x^k (1-x)^{K-k}, \tag{3.2}$$

$\psi^K(x) = \left(b_0^K(x), b_1^K(x), \ldots, b_K^K(x)\right)'$, and $\Pi = (\pi_0, \pi_1, \ldots, \pi_K)'$. The nonparametric function $f(\cdot)$ is now approximated by a linear function of the basis function $\psi^K(x)$ and coefficients Π. Note that the range of x is normalized to $[0,1]$. [15]

One advantage of selecting the Bernstein polynomial as a basis function is the ease of incorporating shape restrictions into the nonparametric function. As highlighted in Section 2.1, the nonparametric income-effect term $f(y-p)$ is weakly increasing. To incorporate this shape restriction within our estimation, we impose constraints on the coefficients Π. With the Bernstein

[15] We standardize the variable $y_{it} - p_{jt}$ that appears in the function $f(\cdot)$ as follows. Given the price data $\{p_{jt}\}_{j\in\{1,\cdots,J_t\},t\in\{1,\cdots,T\}}$ and the simulated draws of income $\{y_{it}\}_{i\in\{1,\cdots,ns\},t\in\{1,\cdots,T\}}$, we define $z_{max} = \max_{i,j,t}\{y_{it} - p_{jt}\}$ and $z_{min} = \min_{i,j,t}\{y_{it} - p_{jt} | y_{it} - p_{jt} > 0\}$. Once done, we define the standardized variable by $z_{ijt} \equiv \frac{y_{it}-p_{jt}}{z_{max}-z_{min}}$, so that the variable z_{ijt} ranges in $[0,1]$ if $y_{it} - p_{jt}$ is positive.

polynomial approximation, the derivative of the function is expressed as:

$$B_K'(x) = K \sum_{k=0}^{K-1} (\pi_{k+1} - \pi_k)\, b_k^{K-1}(x)$$

Thus, the monotonicity restriction (i.e., $B_K'(x) \geq 0$) can be imposed by $\pi_k \leq \pi_{k+1}$ for all k.[16]

Lastly, we need to normalize the function by setting $\pi_0 = 0$, so that we have $f(0) = 0$. This normalization is needed because we cannot identify the level of the income effect term $f(\cdot)$. If $f(\cdot)$ in Equation (2.10) is replaced with $\tilde{f}(\cdot) = f(\cdot) + C$, the constant term C will be canceled out.

3.2 Sieve GMM Estimation with Nested Fixed-Point Algorithm

We incorporate the sieve approximation into the BLP's NFP algorithm to estimate the model parameters. With sieve approximation, the model can be written as:

$$s_{jt} = \int \frac{\mathbf{1}\{y_{it} \geq p_{jt}\} \cdot \exp\left(\psi^K(y_{it} - p_{jt})'\Pi + \beta' X_{jt} + \xi_{jt}\right)}{\exp(\psi^K(y_{it})'\Pi) + \sum_{k=1}^{J_t} \mathbf{1}\{y_{it} \geq p_{jt}\} \cdot \exp\left(\psi^K(y_{it} - p_{jt})'\Pi + \beta' X_{kt} + \xi_{jt}\right)} dG_t(y_{it}). \quad (3.3)$$

Unobserved product characteristics ξ_{jt} represent an econometric error term in the model. The parameters we estimate are summarized by (β, Π).

As is typical in estimating consumer demand models, our model is subject to an endogeneity problem. This concern arises from the correlation between the product price p_{jt} and the unobserved product characteristics ξ_{jt}, which is an econometric error term. Although econometricians lack data on unobserved product characteristics ξ_{jt}, oligopolistic firms may have better access to such information. Consequently, firms may set their product prices by considering product characteristics that are unobservable for econometricians. Such price setting behavior leads to the endogeneity problem associated with product prices.

We address this issue by using IVs. We impose the following conditional mean restrictions:

$$\mathbb{E}\left[\xi_{jt} | Z_{jt}\right] = 0 \text{ for all } j = 1, 2, ...J_t, \text{ and } t = 1, 2, ...T, \quad (3.4)$$

[16] This restriction of the coefficients is a sufficient, but not necessary, condition for the function $f(\cdot)$ to be weakly increasing. Alternatively, the shape restriction could be imposed by adding the constraint that the derivative of $f(x)$ is positive on a grid of points. In Appendix D, we attempt this alternate approach to estimate the demand model in our automobile application. The estimation results with the point-wise restriction are quite similar to that using the restriction on the sieve coefficients. However, this alternate approach requires significantly longer computation times. See Appendix D for details.

where Z_{jt} is a vector of exogenous variables in the utility function (X_{jt}) and additional instruments (W_{jt}). Additional instruments W_{jt} are specified in our empirical application at a later point.

In the estimation, we adopt a sieve generalized method of moments (GMM) estimator (Chen (2007)), which uses the following unconditional moment restrictions:[17]

$$\mathbb{E}\left[\xi_{jt}(\theta)p_b(X_{jt}, W_{jt})\right] = 0, b = 1, \ldots, B, \tag{3.5}$$

where $\theta \equiv (\beta, \Pi)$ denotes a set of model parameters and $\{p_b(X_{jt}, W_{jt})\}_{b=1,\ldots,B}$ is a sequence of known functions that can approximate any real-valued square-integrable functions of X_{jt} and W_{jt} as $B \to \infty$.

The sieve GMM criterion function is given as follows: [18]

$$\xi(\beta, \Pi)'\tilde{\mathbf{P}}(\tilde{\mathbf{P}}'\tilde{\mathbf{P}})^{-}\tilde{\mathbf{P}}'\xi(\beta, \Pi), \tag{3.6}$$

where ξ is a vector that stacks unobserved demand shock ξ_{jt} for $j = 1, \cdots, J_t$ and $t = 1, \cdots, T$. The matrix $\tilde{\mathbf{P}} = [\mathbf{P}, \mathbf{P} \otimes \mathbf{X}]$ denotes a matrix of instruments. We follow Chetverikov et al. (2018) for our choice of matrix $\tilde{\mathbf{P}}$. We first consider a linear span of additional instruments W_{jt} by a known basis function and denote it as $p(W_{jt}) = (p_1(W_{jt}), \cdots, p_B(W_{jt}))'$. The matrix $\mathbf{P}$ is then defined as $\mathbf{P} = (p(W_{11}), \cdots, p(W_{J_T,T}))'$. We also include the tensor product of the columns of two matrices $\mathbf{P}$ and $\mathbf{X} = (X_{11}, \cdots, X_{J_T,T})'$, denoted by $\mathbf{P} \otimes \mathbf{X}$.

In implementing the estimation method introduced above, we must obtain the econometric error term ξ_{jt} given the parameter (β, Π). Since this term nonlinearly enters the demand function (3.3), we use the NFP algorithm to calculate ξ_{jt}. We define the mean utility as $\delta_{jt} = \beta' X_{jt} + \xi_{jt}$, which is the common component of the utility of product j in market t. BLP have shown that there exists

[17] The sieve GMM estimation is considered to be a sieve minimum distance estimation in which the conditional expectation is estimated using a series estimator with an identity weighting matrix (see, e.g., Chen (2007)).

[18] Some of the sieve estimation methods for non- and semiparametric models with endogeneity propose a penalization term on the higher derivative of the nonparametric function to alleviate the ill-posed inverse problem (see, e.g., Chen (2007)). However, in practice, this penalization term is not necessarily used in implementation, as discussed in Chetverikov and Wilhelm (2017). Our approach aligns with this practice for several reasons. First, penalization requires choosing a tuning parameter that governs the strength of penalization. Second, and perhaps more importantly, our approach incorporates a monotonicity restriction on a nonparametric function, which has a similar effect to penalization. In their empirical application, Chetverikov and Wilhelm (2017) demonstrated that the penalization term does not affect the result once a shape restriction has been imposed in the estimation. See Appendix B.3 in Chetverikov and Wilhelm (2017) for further discussion.

a unique vector of $\delta_t = \{\delta_{1,t}, \cdots, \delta_{J_t,t}\}$, such that the observed market share $\{s_{jt}\}_{j=1,\cdots,J_t}$ is equal to the model's predicted market share. The vector of mean utility δ_t can be obtained through a contraction mapping algorithm.

We calculate the value of the objective function given a candidate parameter value of Π as follows: (1) calculate the vector of mean utility δ by applying a contraction-mapping algorithm;[19] (2) run a linear regression of δ on $\mathbf{X}$ to obtain $\hat{\beta}$ and the residual ξ_{jt}; and (3) calculate the value of the objective function (3.6).

We then run a numerical optimization to minimize the objective function. Note that the parameter β appearing in the mean utility function can be obtained by employing a linear GMM.[20] Thus, we need only run a nonlinear optimization routine over Π. This property circumvents the computational costs and allows us to incorporate a rich set of covariates and fixed effects in the mean utility component δ_{jt}.

To calculate the confidence interval of the linear parameter β and the nonparametric function $f(y-p)$, we use a generalized residual bootstrap proposed by Chen and Pouzo (2015) (specifically Theorem 5.2 of their paper).

Adding Random Coefficients While our baseline specification does not incorporate random coefficients on product characteristics x_{jt}, adding them to our framework would be both feasible and straightforward. Specifically, in our estimation method, we can include random coefficient parameters (i.e., standard deviation of random coefficients) into the set of nonlinear parameters along with sieve coefficients Π. To estimate these parameters, we need additional moment conditions to aid identification. Insights from Gandhi and Houde (2019) can be applied, which proposed the use of differentiation instruments. The differentiation IVs are based on the proximity of products in terms of product characteristics.

4 Monte Carlo Simulation

Before applying our demand model to real-world data, we conduct Monte Carlo experiments to assess the efficacy of our methodology. These experiments are designed to demonstrate how well it

[19] We set the tolerance level of the algorithm at 1E-12 .

[20] This estimation trick is called "concentration out." See, e.g., Nevo (2001).

can recover the income effect term, denoted by $f(y-p)$, and the linear coefficients, represented by β in the utility function. Furthermore, we explore the importance of imposing shape restrictions for accurately estimating the non-parametric component.

We further explore a tradeoff inherent to our flexible approach by contrasting our nonparametric estimation method against a parametric method, where the income effect function is a linear one $f(y-p) = \alpha(y-p)$ with parameter α. While our semiparametric approach may be less efficient than the parametric method if the latter precisely captures the true data-generating process (DGP), it notably offers robustness against model misspecification due to its nonparametric nature.

4.1 Data Generating Process

We consider a market t where the total number of products is J_t. We set $J_t = 100$ in our simulations for all market t and $T = 10$. We consider the following utility specification:

$$V_{ijt} = \begin{cases} \beta_0 + \beta_1 x_{jt} + \xi_{jt} + f(y_{it} - p_{jt}) + \epsilon_{ijt} & \text{for } j = 1, \cdots, J \\ f(y_{it}) + \epsilon_{i0t}. & \text{for } j = 0 \end{cases}, \tag{4.1}$$

where ϵ_{ijt} follows an IID type I extreme-value distribution. The observed product characteristic x_{jt} follows the uniform distribution $U(0,1)$. The unobserved product characteristic ξ_{jt} is assumed to follow the normal distribution with mean 0 and standard deviation 0.1 (i.e., $\xi_{jt} \sim N(0, 0.1^2)$.

The product price p_{jt} is given by:

$$p_{jt} = 0.2 + 0.3 x_{jt} + w_{jt} + \xi_{jt}, \tag{4.2}$$

where we include the term ξ_{jt} in the product price to consider the correlation between the price and the unobservable product characteristics.[21] We add the cost shifter w_{jt} following the uniform distribution $U(0,1)$, which serves as an instrument for the product price.

The market share of each product s_{jt} is given by Equation (2.3). To compute s_{jt} in the simulation, we use a numerical integration with a quasi-randomly drawn 1,000 units from the Halton sequence. The income is drawn from the log-normal distribution, i.e., $y_{jt} \sim LN(0, 0.25^2)$.

[21] For simplicity, We assume that the price is competitively determined by the marginal costs in our DGP. We do not incorporate Bertrand competition into the supply side in our Monte Carlo experiments.

Under the DGP, the variable $y - p$ has the support of approximately from -1.5 to 2.5.

The model primitives we estimate are β_0, β_1, and $f(y-p)$. We set $\beta_0 = -5$ and $\beta_1 = 3$, and consider the three different specifications for the income effect term $f(\cdot)$ as follows:

DGP 1 $f(a) = \sinh^{-1}(a)$

DGP 2 $f(a) = \ln(2) + \ln(|a-1|+1)sgn(a-1)$

DGP 3 $f(a) = a$

Note that DGP2 exhibits less smoothness compared to the other two functions because it is not differentiable at $a = 1$. Furthermore, this function demonstrates convexity when $a \in [0,1]$ and concavity for $a \geq 1$. Consequently, after standardizing positive $y - p$ within the range of $[0,1]$, the second-order derivative of the income function changes its sign at approximately the 40th percentile point. For more details on the standardization process, see Footnote 15. It is also crucial to recognize that DGP3 represents the scenario where the parametric model precisely captures the true income function. Thus, in this case, we anticipate that parametric estimation will be more efficient than our proposed nonparametric approach.

4.2 Implementation

We apply both non- and parametric methodologies to datasets generated by each DGP. To nonparametrically estimate the income effect term $f(\cdot)$, we use the sieve approximation method detailed in Section 3.1, employing Kth order Bernstein polynomials. We set several values of K (i.e., $K = 3, 4$, and 5) to assess the impact of different orders on our results. For the parametric approach, we adopt the specification:

$$f(y-p) = \alpha(y-p),$$

where α represents the parameter to be estimated. This specification aligns with the one used in DGP 3.

The cost shifter w_{jt} is used to construct an instrument for the price. More specifically, we set $p(w_{jt}) = (1, w_{jt}, \cdots, w_{jt}^{K-1})'$ as the basis function. Thus, each row of the matrix $\mathbf{P} \otimes \mathbf{X}$ comprises all products of the forms: $p(w_{jt})'x_{jt} = (x_{jt}, w_{jt}x_{jt}, \cdots, w_{jt}^{K-1}x_{jt})$ for all $j = 1, 2, ...J_t$, and $t =$

$1, 2, ...T$. We then apply sieve GMM estimation along with the NFP algorithm.[22] In a parametric method, we set the dimension of the instrumental function to one (i.e., $p(w_{jt}) = (1, w_{jt})$).

To measure the estimation precision for the nonparametric income effect term, we compute the mean integrated squared error (MISE) by $MISE = \frac{1}{NS}\sum_{r=1}^{NS}\left(\int_0^1 \left(f(z) - \hat{f}_r(z)\right)^2 dz\right)$, where z is the standardized value of $y - p$ ranging from 0 to 1 and $\hat{f}_r(z) = \widehat{\alpha_r} z$ for parametric estimation.[23] The subscript r denotes the index for a simulation. The total number of simulations NS is set to 100. Regarding the linear parameters (β_0, β_1), we calculate the mean bias and root mean square error (RMSE) given by $Bias(\beta_j) = \frac{1}{NS}\sum_{r=1}^{NS} \hat{\beta}_j^r - \beta_j$ and $RMSE(\beta_j) = \frac{1}{NS}\sum_{r=1}^{NS}(\hat{\beta}_j^r - \beta_j)^2$.

4.3 Simulation Results

Simulation results for DGP1-3 are reported in Table 1, Table 2, and Table 3, respectively. Each table reports the MISE of the nonparametric function $f(y - p)$ and the bias and RMSE of linear parameters (β_0, β_1).

The first observation is that simulations incorporating shape restrictions significantly outperform those without such restrictions. Across all DGPs, the MISE of the nonparametric function $f(y-p)$ is considerably reduced when a shape restriction is applied to the income effect term. Similarly, the RMSE of the linear parameters (β_0, β_1) also shows improvement with shape restriction. The impact of shape restriction is visually evident in Figures 1, 3, and 5, which display the 95% confidence intervals (CI) for estimations both with and without shape restrictions. Without shape restriction, the estimate is imprecise near the support's endpoint, whereas the confidence band remains tighter under shape restriction. Our findings align with those of Chetverikov and Wilhelm (2017), who found significant performance enhancements from applying a monotonicity restriction in a semiparametric partially linear model with endogeneity. Our simulations further extend their conclusion to scenarios where the model is non-separable.

We now evaluate the performance of our nonparametric approach against the parametric model, which assumes $f(y-p) = \alpha(y-p)$. In DGP1, the MISE is significantly lower for the nonparametric estimation with shape restriction compared to the parametric method. The left panel of Figure 2 illustrates that our nonparametric method accurately estimates the shape of the income effect func-

[22] We set the tolerance level of the NFP algorithm as 1E-12 and employ the constrained minimization procedure in the Knitro solver.

[23] The integral inside the parentheses are computed using Monte Carlo integration.

tion, whereas the parametric estimation incurs bias from model misspecification. Intriguingly, and as indicated in Table 2, the MISE for $f(y-p)$ in DGP2 is higher using the nonparametric approach than the parametric model. This is because, as demonstrated in Figure 4, the misspecification bias is minimal, given that the shape of $f(y-p)$ in DGP2 closely resembles a linear function. Furthermore, the 95% CI in the nonparametric method is broader than that in the parametric method, as depicted in the right panel of Figure 4. Lastly, in DGP3, where the parametric model aligns correctly with the true data-generating process, the MISE is significantly lower in the parametric approach than in the nonparametric one.

These results illustrate the inherent tradeoff of our nonparametric approach. On the one hand, it excels in accurately estimating the shape of the income effect function, even when deviating from linearity. This capability is particularly valuable in empirical contexts where the precise form of the income effect function is unknown. However, a common drawback of nonparametric estimation is the broader CI associated with the estimated function, reflecting the cost of its flexibility.

Table 1: Results of Monte Carlo Simulations under DGP 1

(i) MISE of $f(y-p)$

	Without SR	With SR	Parametric
$K=3$	0.0268	0.0025	0.0099
$K=4$	0.0142	0.0050	
$K=5$	0.1075	0.0069	

(ii) Estimation of β_0

	Without SR		With SR		Parametric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	0.0058	0.1032	-0.0043	0.0249	0.0074	0.0097
$K=4$	-0.0072	0.0838	0.0078	0.0405		
$K=5$	0.0143	0.1557	-0.0024	0.0482		

(iii) Estimation of β_1

	Without SR		With SR		Parameric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	0.0018	0.0122	0.0002	0.0109	0.0026	0.0107
$K=4$	0.0026	0.0123	0.0005	0.0110		
$K=5$	0.0009	0.0123	0.0000	0.0128		

Note: SR is an abbreviation of shape restriction.

Table 2: Results of Monte Carlo Simulations under DGP 2

(i) MISE of $f(y-p)$

	Without SR	With SR	Parametric
$K=3$	0.0289	0.0039	0.0018
$K=4$	0.0192	0.0046	
$K=5$	0.0526	0.0049	

(ii) Estimation of β_0

	Without SR		With SR		Parametric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	-0.0375	0.1053	-0.0184	0.0315	-0.0253	0.0091
$K=4$	-0.0070	0.1033	-0.0219	0.0390		
$K=5$	0.0238	0.1198	-0.0185	0.0440		

(iii) Estimation of β_1

	Without SR		With SR		Parameric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	-0.0012	0.0114	-0.0009	0.0111	-0.0011	0.0108
$K=4$	-0.0009	0.0124	-0.0003	0.0113		
$K=5$	0.0015	0.0121	-0.0005	0.0123		

Note: SR is an abbreviation of shape restriction.

Table 3: Results of Monte Carlo Simulations under DGP 3

(i) MISE of $f(y-p)$

	Without SR	With SR	Parametric
$K=3$	0.0356	0.0044	0.0002
$K=4$	0.0207	0.0065	
$K=5$	0.0615	0.0066	

(ii) Estimation of β_0

	Without SR		With SR		Parametric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	-0.0123	0.1140	0.0038	0.0376	0.0010	0.0105
$K=4$	0.0098	0.0981	-0.0050	0.0496		
$K=5$	0.0085	0.1249	-0.0036	0.0519		

(iii)Estimation of β_1

	Without SR		With SR		Parameric	
	Bias	RMSE	Bias	RMSE	Bias	RMSE
$K=3$	0.0010	0.0116	-0.0002	0.0121	-0.0017	0.0114
$K=4$	0.0010	0.0107	0.0013	0.0108		
$K=5$	0.0004	0.0138	0.0006	0.0112		

Note: SR is an abbreviation of shape restriction.

Figure 1: The Role of Shape Restriction under DGP 1

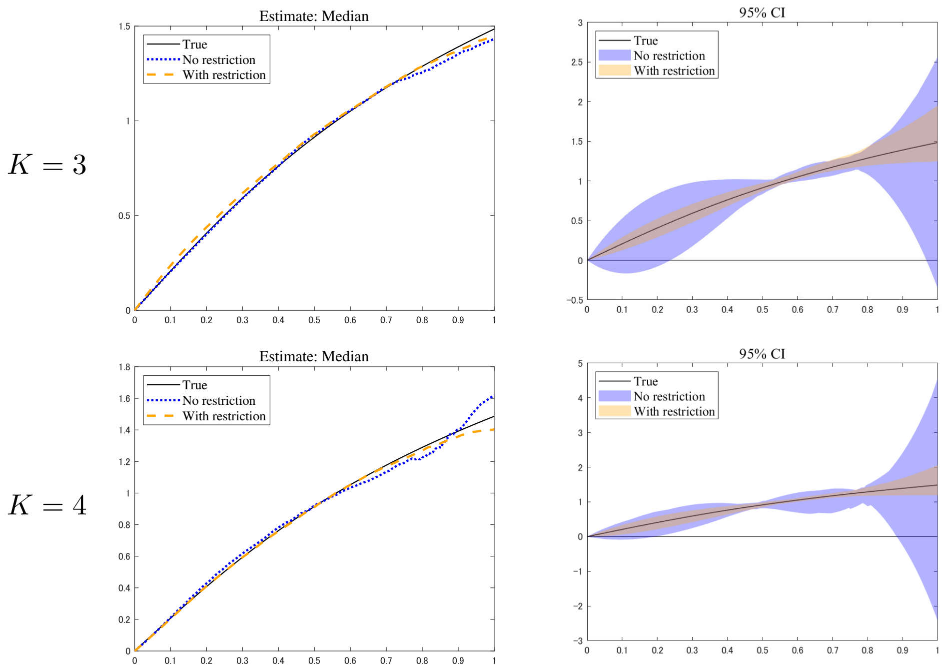


Figure 2: Nonparametric and Parametric Estimation under DGP 1

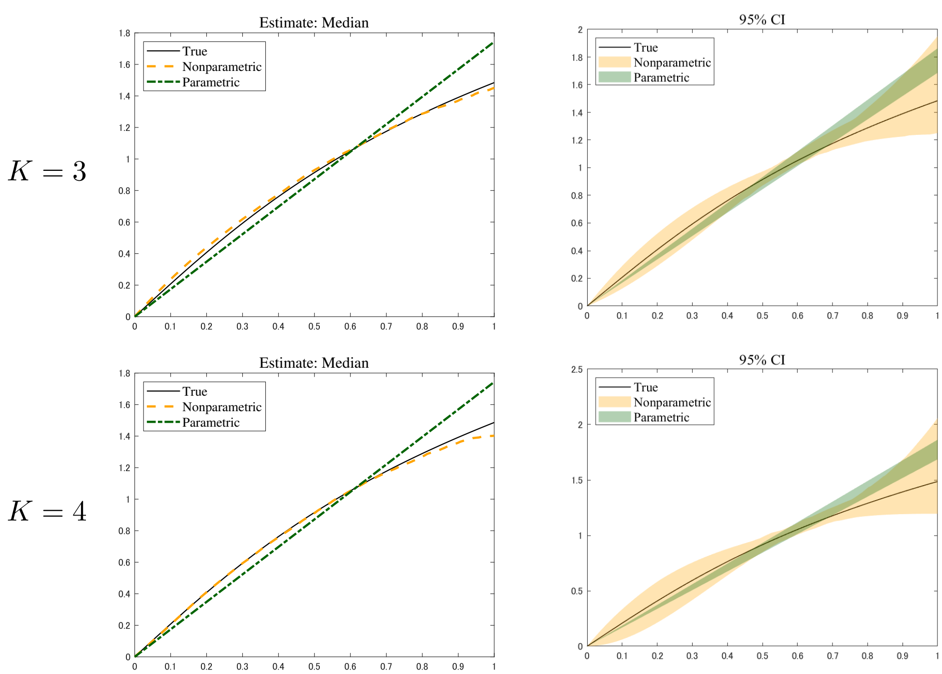


Figure 3: The Role of Shape Restriction under DGP 2

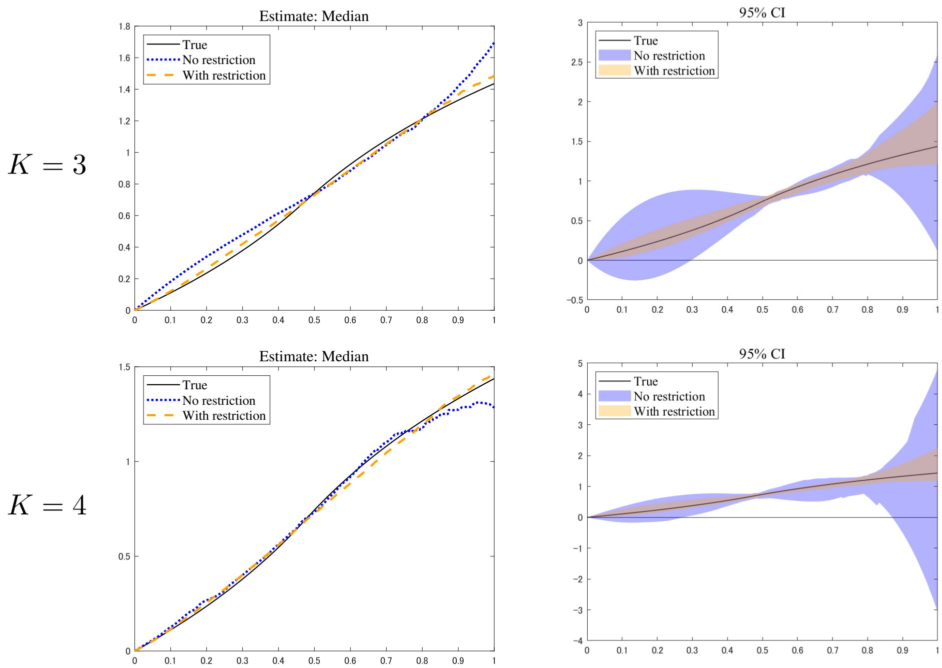


Figure 4: Nonparametric and Parametric Estimation under DGP 2

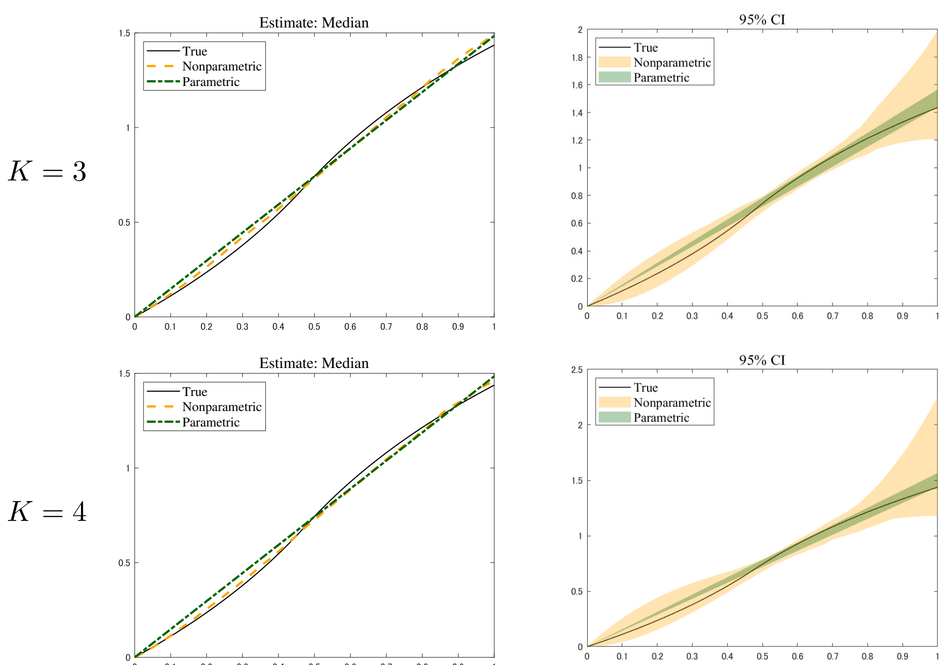


Figure 5: The Role of Shape Restriction under DGP 3

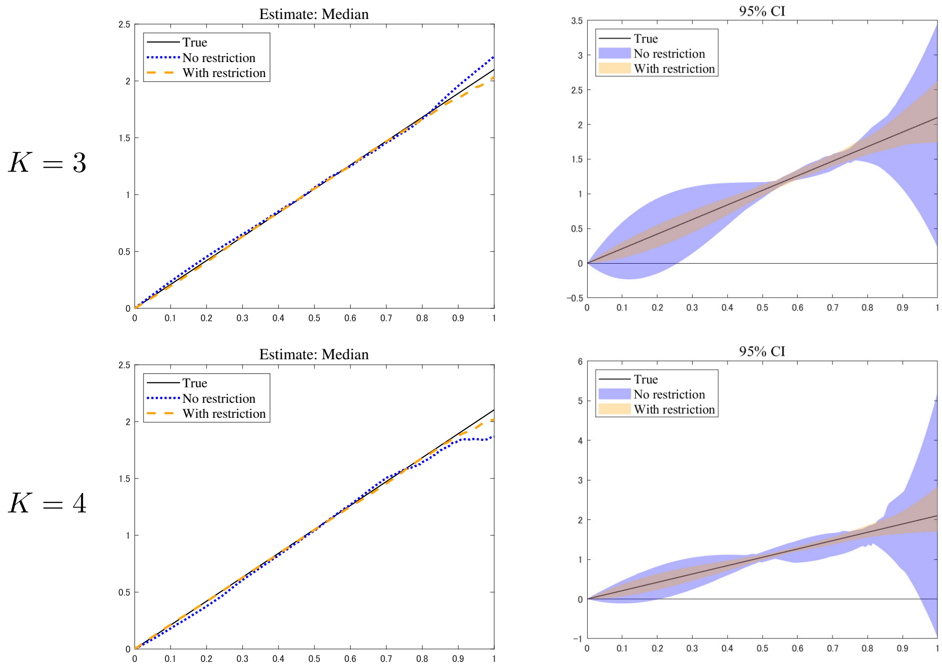
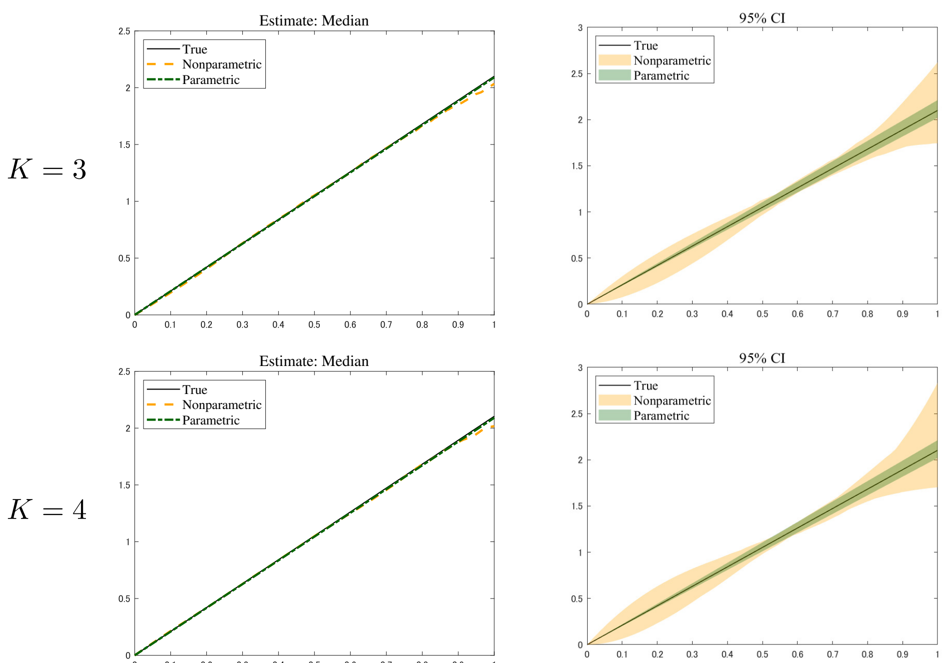


Figure 6: Nonparametric and Parametric Estimation under DGP 3



5 Empirical Application: Automobile Demand

We now apply our semiparametric demand model to the Japanese automobile market to demonstrate the practical relevance of our framework. In Section 5.1, we briefly explain the data and institutional background. Preliminary results, estimated using the linear logit model by Berry (1994), are presented in Section 5.2, where we also discuss the limitations of this approach. To address these limitations, we introduce the semiparametric specification of the demand model in Section 5.3. The estimation results of the semiparametric model are discussed in Section 5.4. Building on these results, we conduct counterfactual simulations detailed in Section 6, evaluating the implications of a feebate policy for eco-friendly cars in Japan and the potential impact of a hypothetical merger between two leading manufacturers: Toyota and Honda.

5.1 Data

Two types of datasets are constructed. The first contains information on the Japanese automobile market between 2006-2013, including product-level information on sales, prices, and product characteristics for each year. Second, we construct the income distribution of Japanese households as a source of consumer heterogeneity in the demand model (see Appendix B for details).

To construct the former dataset on the Japanese automobile market, we combine the catalog information of car models and the registration of newly-purchased cars.[24] The dataset is an unbalanced panel at the model-and-year level.

We define the share of each car model in each year (s_{jt}) as the fraction of the total number of new car registrations (see Footnote 24) over the total number of households in Japan, sourced from annual reports of *Population, demographics, and the number of households based on the Basic Resident Register* conducted by the Ministry of Internal Affairs and Communications.[25] The share of households not purchasing any automobile is defined as $s_{0t} = 1 - \sum_{j \in J_t} s_{jt}$, where J_t represents

[24] The catalog information is obtained from the website *CarView!*, which provides the specifications of car models and their list prices. Registrations of standard and compact cars are obtained from the *Annual Report of New Car Registrations* (*Shinsha Touroku Daisu Nempou*) issued by the Japan Automobile Dealers Association. Registrations for minicars are obtained from a report published by the Japan Mini Vehicle Association (see `https://www.zenkeijikyo.or.jp/statistics/tushokaku`; in Japanese. Accessed on January 6, 2023.). Finally, we source information on registrations for 20 top-selling imported cars from a report published by the Japan Automobile Importers Association (see `http://www.jaia-jp.org/english-transition/`. Accessed on January 6, 2023).

[25] See `https://www.soumu.go.jp/main_sosiki/jichi_gyousei/daityo/jinkou_jinkoudoutai-setaisuu.html` for details (In Japanese. Accessed on January 6, 2023).

the set of car models available in market t (i.e., year t).

The observable product characteristics (X_{jt}) include (1) the ratio of horsepower to the weight of the car (HP/WT), (2) car size (Size), (3) fuel efficiency (miles per gallon [MPG]), and (4) a dummy variable that indicates whether the model has an automatic/continuously variable transmission system (AT/CVT).[26] We also use dummy variables for mini, foreign, and hybrid cars.[27]

The data also include the list price p_{jt} offered by manufacturers. We construct an effective price p^e_{jt} that reflects the tax and subsidy. The effective price p^e_{jt} is defined as follows:

$$p^e_{jt} = (1 + \rho_{jt})p_{jt} + T_{jt} - ES_{jt}, \tag{5.1}$$

where ρ_{jt} is the rate of the ad-valorem tax, including consumption tax (5% during the sample period), T_{jt} is the specific tax, and ES_{jt} is a subsidy for eco-friendly cars. All prices and taxes are deflated by the 2015 Consumer Price Index (CPI).

A notable feature of the Japanese automobile market that plays an important role in the analysis is the presence of various tax and subsidy policies. The Japanese government introduced a feebate policy named *Eco-car Subsidy (ES)* program in 2009 as part of the economic stimulus measures in the wake of the Great Recession. Table 4 provides an overview of the policy.[28] The program has two phases. The first phase of the ES program was effective from April 2009 to September 2010.[29] In this phase, cash rebates of JPY $100,000$ (approximately USD 1,000) and JPY $50,000$ (approximately USD 500) were offered to normal cars and minicars that exceeded the 2010 fuel efficiency standard by 15%, respectively.[30] The second phase began in December 2011 and continued until January 2013. In the second phase, the eligibility to receive a cash rebate was made stricter than in the first. JPY 100,000 and JPY 70,000 were subsidized to normal cars and minicars exceeding the 2015 fuel efficiency standard, which is equivalent to 125% of the 2010 standard.

Table 5 presents the descriptive statistics of our dataset. During the sample period, there was a substantial decrease (approximately 24%) in the total tax burden. The average ES subsidy

[26] Given that the fuel efficiency is measured in kilometers per liter, we convert it to miles per gallon using mpg = (fuelefficiency/1.60934) × 3.78541.

[27] A minicar is a category of automobile with a length of 3.4 meters or less, a width of 1.48 meters or less, and a height of 2.0 meters or less, as well as a displacement level of 660 cc or less.

[28] The details of the tax policy are relegated to Appendix C.

[29] In the first phase, consumers had the option to apply for the ES program with a higher amount of subsidy conditional on scrapping their existing vehicle if it was older than 13 years. See Kitano (2022) for details.

[30] We use an exchange rate of 100 JPY/USD throughout the paper.

Table 4: Details of the Eco-car Subsidy

	Phase 1	Phase 2
Period	April 2009 to September 2010	December 2011 to January 2013
Subsidy to normal cars	JPY 100,000	JPY 100,000
Subsidy to minicars	JPY 50,000	JPY 70,000
Requirement	exceeding the 2010 fuel efficiency standard by 15%	exceeding the 2015 fuel efficiency standard

amounted to JPY 19,000 between 2009 and 2013. In terms of automobile characteristics, we observe an improvement in the average MPG (fuel efficiency), while no significant changes are detected in other characteristics. Although a slight increase in effective prices can be noted following the subsidy's introduction, this change is primarily attributable to variations in product characteristics. Table A1 in Appendix A shows a regression analysis of effective prices on the subsidy amount and taxes, along with product characteristics. This analysis shows that both the subsidy and taxes influenced prices as anticipated: the former led to lower prices, while the latter resulted in higher prices.

Table 5: Descriptive Statistics of Japanese Automobile Market Data

	(1) 2006-2008				(2) 2009-2013			
	Mean	SD	Min	Max	Mean	SD	Min	Max
$\log(s_{jt}/s_{0t})$	-8.446	1.563	-13.766	-5.347	-8.860	1.782	-15.435	-5.052
p^e_{jt} (JPY 1 Million)	2.731	1.966	0.780	12.870	2.772	1.973	0.771	13.946
Total Tax (JPY 1 Million)	0.186	0.113	0.030	0.682	0.144	0.120	0.008	0.707
ES (JPY 1 Million)	0.000	0.000	0.000	0.000	0.019	0.039	0.000	0.104
HP/WT	0.099	0.033	0.047	0.276	0.099	0.036	0.045	0.318
MPG	34.595	10.600	12.937	83.501	37.142	11.974	15.524	83.266
Car Size	7.485	0.676	6.115	8.855	7.520	0.673	6.115	8.825
AT/CVT	0.978	0.148	0.000	1.000	0.987	0.115	0.000	1.000
Minicar Dummy	0.202	0.402	0.000	1.000	0.198	0.399	0.000	1.000
Hybrid Car Dummy	0.008	0.090	0.000	1.000	0.042	0.201	0.000	1.000
Observations	495				827			

Notes: s_{jt} and s_{0t} represent the market share of product j and outside good in market t . p^e_{jt} indicates the effective price that consumers face. "Total Tax" denotes the sum of automobile acquisition tax, automobile weight tax, and automobile tax. "ES" is an abbreviation of "eco-car subsidy". All the price and tax variables are deflated by the 2015 CPI. Product attribute variables include the ratio of horsepower to car weight (HP/WT), millage per gallon (MPG), car size (Size), dummy variables indicating whether the model has an automatic or continuously variable transmission (AT/CVT Dummy), and minicar and hybrid car (minicar and hybrid car dummy).

5.2 Preliminary Results from a Linear Logit Model

As a benchmark, we first estimate the parametric version of the demand model. Specifically, we estimate the quasi-linear specification given by $f(y-p) = \alpha(y-p)$, where α is a parameter to be estimated. Given that the income term y is canceled out, the model is reduced to the linear logit model of Berry (1994) as follows:[31]

$$\ln\left(\frac{s_{jt}}{s_{0t}}\right) = \alpha p^e_{jt} + \beta' X_{jt} + \theta_{f(j)} + \theta_t + \xi_{jt}, \tag{5.2}$$

where s_{jt} is the market share of the automobile j at year t, s_{0t} is the market share of outside option (i.e., not purchasing any car) at year t, p^e_{jt} is the effective price of the automobile j at year t, and the vector X_{jt} includes HP/WT, MPG, car size, AC/CVT dummy, minicar dummy, and hybrid car dummy. Furthermore, the firm fixed effects ($\theta_{f(j)}$) and year (market) fixed effects (θ_t) are controlled. ξ_{jt} stands for the econometric error term.

We estimate this model using ordinary least squares (OLS) and two-stage least squares (2SLS), wherein the set of IVs are employed to handle the endogeneity of effective price p^e_{jt}. We construct the instruments based on tax and subsidy policy by following Konishi and Zhao (2017) and Kitano (2022).[32] Specifically, our instruments are defined by (1) the sum of the tax amount of other products produced by the firm $\sum_{k\in J_f, k\neq j}(Tax)_{kt}$ and (2) the sum of the tax amount of competitors' products $\sum_{k\notin J_f}(Tax)_{kt}$. Note that Tax_{jt} represents the sum of automobile weight tax and automobile tax of car model j in year t. Importantly, the automobile acquisition tax, which depends on the automobile price and is thus endogenous, is not included in the variable Tax_{jt}.

We now turn to the validity of our chosen instruments. The relevance of the instruments is supported by the fact that automobile manufacturers account for the tax rate when setting prices. We further argue for the relevance based on the first-stage regression in Table 6. Concerning the independence of the instruments, the tax rate should be uncorrelated with the unobserved characteristics ξ_{jt}. However, a potential challenge arises if automobile manufacturers strategically respond to changes in the tax rate, possibly by introducing more eco-friendly cars eligible for tax reductions. This strategic behavior could compromise our instruments' validity, particularly if firms anticipate unobserved demand shocks (ξ_{jt}) in the timing of product introduction.

[31] More precisely, we also omit the budget constraint in order to derive Berry (1994)'s linear logit model. Even if the quasi-linear specification is assumed in our model, the presence of the budget constraint is a source of consumer heterogeneity. Thus, the budget constraint does not allow us to use a linear regression model, as in Berry (1994).

[32] We also use differentiation instruments proposed by Gandhi and Houde (2019) to assess its performance. Moreover, we attempted to use traditional BLP instruments for car characteristics, though the first-stage regression of a parametric IV logit model was substantially weaker than differentiation IV. The result of using traditional BLP IV is not reported. This finding is consistent with results obtained by Konishi and Zhao (2017) (see Appendix D in their paper).

To address this potential issue, we follow Eizenberg (2014). Our key identification assumption is that firms do not observe the demand shock ξ_{jt} prior to product introduction. This assumption implies that product selection is influenced by observable automobile characteristics rather than the unobserved demand shock. We can mitigate the above concern by controlling for these observable characteristics that may impact product introduction. In our analysis, car types (e.g., mini, foreign, and hybrid cars) are related to the eligibility for tax reductions and ESs. Consequently, our utility function includes dummy variables for these car types in addition to standard product characteristics.[33]

Table 6 shows the estimation results of Equation (5.2). Column (1) reports the results of OLS estimation, where the price is treated as an exogenous attribute. The results of the IV estimation are then summarized in columns (2)-(4). Estimated coefficients have the expected signs. By comparing the results of OLS and IV estimation, the price coefficient seems to be underestimated in the former (i.e., the OLS estimate is biased toward zero). Regarding the validity of the instruments, the result shown in column (3) suggests that the tax-based instrument has a high value of Kleibergen-Paap F statistics in the first stage with notably low J statistics. Based on the above results, we use a tax-based instrument to estimate the semiparametric demand model. We also use column (3) as a parameter of the linear logit model in the simulation analysis reported in Section 6.

While the estimation of the linear logit model is useful as a preliminary analysis, the model has several issues in practice. First, as discussed in Section 2.4, the quasi-linear assumption imposes a strict restriction on the pattern of price elasticity. The relationship between the own-price elasticity and price is linear under a simple logit model (we revisit this point when estimating the price elasticity in a semiparametric demand model). In addition, since the demand curve is always log-concave in price under a simple logit model, the pass-through rate is bounded by 1 (Weyl and Fabinger 2013). Such property may imply an underestimation of the pass-through rate of the subsidy.

[33] While the assumption of exogenous product characteristics may appear strong, it is, in fact, the conventional approach in the estimation of differentiated product demand. Given our emphasis on demand estimation and its implications for short-run pricing decisions, we employ this assumption to simplify the analysis. See, e.g., Barwick et al. (2024) for an analysis that considers endogenous product attributes.

Table 6: Estimation Results of Linear Logit Model

	(1)	(2)	(3)	(4)
	OLS	IV	IV	IV
Effective Price (p^e_{jt})	-0.402	-0.646	-0.929	-0.703
	(0.030)	(0.119)	(0.153)	(0.105)
HP/WT	5.898	13.598	22.550	15.424
	(1.578)	(3.830)	(4.853)	(3.429)
MPG	0.105	0.100	0.093	0.098
	(0.006)	(0.007)	(0.007)	(0.007)
Car Size	1.686	1.976	2.314	2.045
	(0.115)	(0.163)	(0.189)	(0.148)
AT/CVT	0.257	0.437	0.647	0.480
	(0.385)	(0.385)	(0.399)	(0.385)
Differentiation IV on car attribute	No	Yes	No	Yes
Tax-based IV	No	No	Yes	Yes
Kleibergen–Paap F statistics	NA	21.650	18.290	16.316
Hansen J statistics	NA	25.702	2.614	30.986
Observations	1,322	1,322	1,322	1,322

Notes: All regression includes year fixed effects, firm fixed effects, minicar dummy and hybrid car dummy. The robust standard error is reported in parentheses.

5.3 Semiparametric Demand Specification

We now take the semiparametric approach introduced in Section 2 to overcome the potential issues of the simple logit model. The following specification is considered for the conditional indirect utility:

$$V_{ijt} = f(y_{it} - p^e_{jt}) + \beta' X_{jt} + \theta_{f(j)} + \theta_t + \xi_{jt} + \epsilon_{ijt}, \ (\forall j \in \{1, \cdots, J_t\}) \tag{5.3}$$

$$V_{i0t} = f(y_{it}) + \epsilon_{i0t} \tag{5.4}$$

where $f(\cdot)$ is a weakly-increasing continuous function, y_{it} stands for the real income for individual i at year t, and ϵ_{ijt} is idiosyncratic shock following Type-I extreme value distribution. The definition of all the other components is the same as in Equation (5.2). We employ the tax-based IV defined in the previous section (i.e., $w_{1,kt} \equiv \sum_{k \in J_f, k \neq j} (Tax)_{kt}$, and $w_{2,kt} \equiv \sum_{k \notin J_f} (Tax)_{kt}$) to estimate $f(\cdot)$ and β in the model. Specifically, the matrix of IVs defined in equation (3.6) is based on $p(w) = (1, w_{1,kt}, w_{2,kt}, w^2_{1,kt}, w^2_{2,kt}, w^3_{1,kt}, w^3_{2,kt})$, and the tensor products of $p(w)$ and X_{jt}.

The income effect term $f(y-p)$ is approximated by the following specification:

$$f(y-p) = \frac{-1}{\sum_{k=1}^{K} \pi_k b_k^K (y-p)},$$

where $b_k^K(x)$ is a Bernstein polynomial defined in (3.2). We set $K = 4$ in estimation.[34] Regarding the shape restriction, we impose the monotonicity restriction on a Bernstein polynomial by $\pi_k \leq \pi_{k+1}$ for $k = 1, \cdots, K-1$, so that the income effect term $f(y-p)$ is a weakly-increasing function.

The approximation uses the inverse of a Bernstein polynomial, which differs from the regular Bernstein polynomial used in Monte Carlo simulations. This approach ensures that $\lim_{x \to 0} f(x) = -\infty$, a restriction that, although seemingly unnecessary, is essential for maintaining the continuity of the demand model in relation to prices. Such continuity is pivotal for accurately simulating a pricing equilibrium in numerical analyses. We elaborate on the rationale behind this choice below.

Recall that the individual choice probability $s_{ijt}(\cdot)$, defined in Equation (2.10). Due to the budget constraint in the utility maximization problem (i.e., $\mathbf{1}\{p_{jt} \leq y_{it}\}$), this choice probability can be discontinuous in terms of the price at $p_{jt} = y_{it}$. Specifically, the choice probability s_{ijt} is positive as long as $p_{jt} < y_{it}$, but becomes zero ($s_{ijt} = 0$) when $p_{jt} > y_{it}$. Given that we approximate aggregate demand by simulating a finite number of consumers, the aggregate demand function has a range of discontinuous points. While such discontinuity is not detrimental to estimation where prices are fixed as a covariate, it makes the supply-side analysis (i.e., solving the Bertrand pricing equilibrium) numerically challenging.

This issue can be avoided by imposing a restriction whereby $\lim_{p \to y} f(y-p) = -\infty$. Under this restriction, the individual choice probability $s_{ijt}(\cdot)$ converges to zero as the price approaches the income level. This property ensures the continuity of the demand function. In Appendix E, we graphically demonstrate this point using a numerical example.

The aggregate demand is determined by integrating the individual choice probability with income distribution. We assume the income distribution to be the log-normal distribution with mean

[34] Figure A1 in Appendix A reports the results of a robustness analysis concerning the choice of K. The results indicate that the estimated own-price elasticities remain similar across the choice of $K = 3, 4$, and 5. Although a data-driven method for determining K would be preferable, current theoretical guidance on selecting the order of sieve polynomials (K in our context) for general nonparametric models with endogeneity is limited, as noted by Chen and Qiu (2016). Chen et al. (2023) recently introduced a data-driven approach to determine the sieve dimension, considering models with conditional moment restrictions of the form $E[Y - h_0(X)|W] = 0$, where Y is the dependent variable, X is an endogenous variable, W is a set of instruments, and $h_0(\cdot)$ is the nonparametric function to be estimated. However, our model falls outside the scope of the models addressed in their analysis.

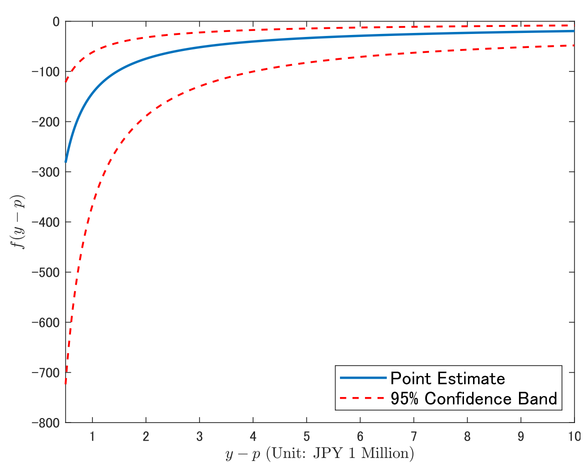
μ_t and standard deviation σ_t: $LN(\mu_t, \sigma_t^2)$. We estimate these two parameters (μ_t, σ_t) for each year using data from *the Comprehensive Survey of Living Conditions*. See Appendix B for the details. To numerically compute the integral, we use quasi-random sampling. We draw 1,000 consumers from the estimated distribution by a Halton sequence.

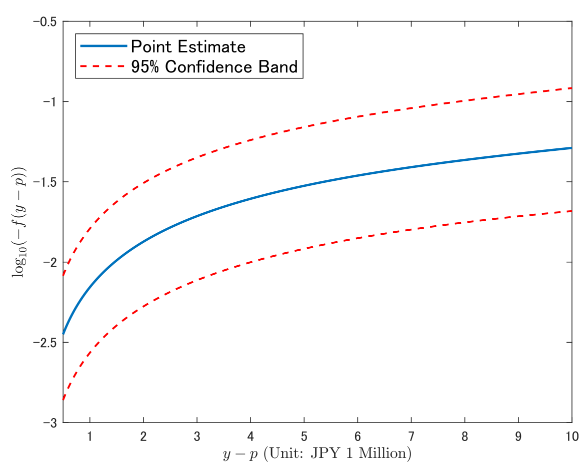
5.4 Estimation Results of Semiparametric Model

Figure 7 and Table 7 report estimates of the income-effect term $f(y-p)$ and linear parameters β, respectively. Figure 7 shows that the income effect term $f(y_{it}-p_{jt})$ is nonlinear and concave. The marginal utility from the disposable income after purchasing an automobile is higher for low-income households. Table 7 reports the estimation results of linear parameters in Equation (5.3). The point estimates are comparable with the simple logit model, except for the HP/WT, for which the coefficient is approximately 1.5 times larger when we employ the semiparametric estimation (see column (3) of Table 6). However, the CIs of linear parameters in the semiparametric models are broader than those in the linear logit model. This result suggests that flexible estimation of the income effect affects the precision of other parameter estimates, demonstrating a trade-off that must be considered when choosing the estimation method.

Figure 7: Estimation of $f(y_{it}-p_{jt})$

(i) $f(y-p)$ is plotted. (ii) $-\log_{10}(-f(y-p))$ is plotted.





Note: The figure the reports point estimate and 95% confidence band based on 200 times bootstrap sampling. The range of $y-p$ plotted is JPY 0.5 to 10 Million.

Based on the estimated demand function, we calculate the own-price elasticity. Figure 8 plots

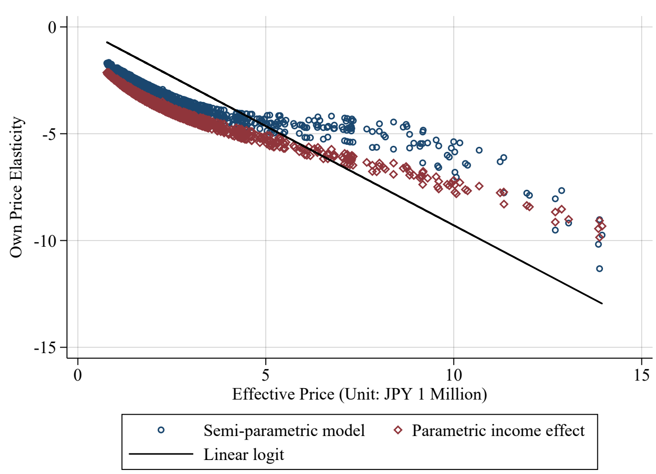
Table 7: Estimation of Linear Parameters

	Estimate	95% CI
Constant	-25.147	[-27.587, -21.078]
HP/WT	21.315	[10.744, 33.838]
MPG	0.077	[0.042, 0.098]
Car Size	2.508	[1.877, 3.324]
AT/CVT	0.269	[-0.676, 1.531]
Observations		1,322

Note: The point estimate of selected linear parameters and 95% confidence interval are reported. The confidence interval is constructed using 200 bootstrap samples. See Theorem 5.2 of Chen and Pouzo (2015) for the details. Minicar dummy, hybrid car dummy, year fixed effects, and firm fixed effects are also included but not reported.

the estimated own-price elasticity and the effective price of automobiles for each product. We compare the own-price elasticity based on our semiparametric model (blue circle) with that from a simple logit model (black line). As discussed in Section 5.2, the simple logit model implies a linear relationship between elasticity and price (see Equation (2.11) in Section 2.4). However, our semiparametric model reveals a nonlinear relationship where the estimated elasticity is relatively constant in the range of JPY 2 to 10 million. Consequently, the elasticity estimated by simple logit suffers from underestimation for inexpensive cars (e.g., minicars) and from overestimation for luxury cars.

Figure 8: Estimated Own-Price Elasticity



In addition to the linear logit model, we estimate the following parametric specification with consumer heterogeneity.

$$
\begin{aligned}
V_{ijt} &= \frac{\alpha}{y_i} p^e_{jt} + \beta' X_{jt} + \theta_{f(j)} + \theta_t + \xi_{jt} + \epsilon_{ijt}, &(5.5)\\
V_{i0t} &= \epsilon_{i0t} &(5.6)
\end{aligned}
$$

The price coefficient is inversely proportional to the income level y_{it}, and is thus heterogeneous across consumers. Note that this specification is the same as Berry et al. (1995) and Berry et al. (1999), and is considered a first-order approximation of the parametric income effect of $f(y_{it} - p_{jt}) = \alpha \log(y_{it} - p_{jt})$.[35]

The estimated price elasticity in a parametric BLP model (red diamond) is also plotted in Figure 8. The overall pattern departs from the simple logit model and is close to the one from the semiparametric model. However, given the price level, the estimated own-price elasticity is more heterogeneous in the semiparametric model than the parametric BLP model.

To further examine the difference across specifications, we plot the relationship between the own-price elasticity (i.e., the first derivative) and the demand curvature (i.e., the second derivative) in Figure 9. Following Mrázová and Neary (2017) and Miravete et al. (2023), we calculate the curvature of demand ρ_j for product j by

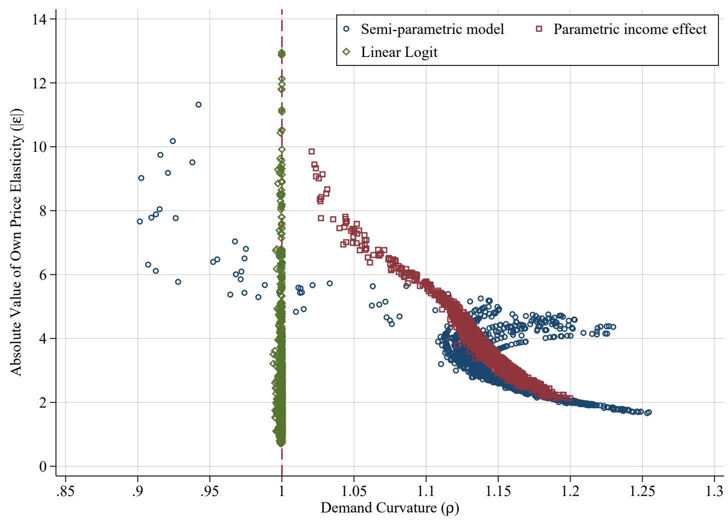
$$
\rho_j = \frac{q_j \cdot \frac{\partial^2 q_j}{\partial p_j^2}}{\left(\frac{\partial q_j}{\partial p_j}\right)^2}.
$$

Figure 9 shows that the curvature estimates in the linear logit model (green diamond) are bounded by, and concentrated at, one. On the other hand, the other two models (i.e., parametric income effect in red square and semiparametric model in blue circle) exhibit the heterogeneity in the curvature estimates. In particular, our semiparametric model shows the most significant heterogeneity

[35] While we could incorporate random coefficients on product characteristics X_{jt}, we decided not to do so for several reasons. First, when we attempted to estimate the specification with random coefficients on product characteristics by exploiting the differentiation instruments, the estimated parameters for the standard deviation of random coefficients were imprecise and close to zero. This could have been due to the limited number of markets in our application (i.e., $T = 8$), which made it difficult to estimate random coefficient parameters. Secondly, the baseline specification of the semiparametric model does not incorporate random coefficients. Our objective is to contrast the baseline model with the parametric specification of the income effect term, thereby highlighting the advantages of the nonparametric approach.

in the curvature estimates. This is because the curvature of our semiparametric model depends on the second-order derivative of the income effect function $f''(y-p)$, which we flexibly estimate via a sieve method.[36]

Figure 9: Relationship between Own-price Elasticity and Demand Curvature



6 Policy Simulations: Pass-Through and Merger Analysis

In this section, we conduct two counterfactual simulations using our estimated semiparametric model. The first analyzes the ES program-particularly the pass-through of the subsidy and its welfare impacts. As discussed in Section 2.4, our demand model can flexibly capture the curvature of the demand function, which has key implications for the pass-through. Second, we conduct a merger simulation between two automobile manufacturers. In both simulations, we compare the results from different demand specifications, namely (1) semiparametric model, (2) parametric random coefficient model, and (3) linear logit model. Below, we introduce a supply model in Section 6.1 to estimate the marginal costs of car models and simulate counterfactual equilibria. We then discuss results from counterfactual simulations in Sections 6.2 and 6.3.

[36] More precisely, the second-order derivative of the demand is given by

$$\frac{\partial^2 s_j}{\partial p_j^2} = \int \left\{ s_{ij}(1-s_{ij})(1-2s_{ij}) \left[f'(y-p_j)\right]^2 + s_{ij}(1-s_{ij}) f''(y_i - p_j) \right\} dG(y_i).$$

The second-order derivative term $f''(y_i - p_j)$ is equal to zero in a parametric random coefficient model.

6.1 Supply Model

We consider a model of Bertrand competition with differentiated products, as in BLP and Nevo (2001). Automobile manufacturers are multiproduct oligopolists that compete in prices. The profit for manufacturer f in year t is given as follows:

$$\pi_{ft} = \sum_{j \in \mathbf{J}_{ft}} (p_{jt} - mc_{jt}) q_{jt} \left(\mathbf{p}_t^e\right), \tag{6.1}$$

where mc_{jt} is the marginal cost of car model j in year t. We assume a constant marginal cost. The variable $\mathbf{p}_t^e$ is a vector of effective prices in market t and defined as $\mathbf{p}_t^e = \{p_{jt}^e\}_{j \in J_t}$. Note that we distinguish between the price charged by a firm (i.e., p_{jt}) and the effective price (i.e., p_{jt}^e) that reflects the tax and subsidy. It should be remembered that the effective price is given by $p_{jt}^e = (1+\rho_{jt})p_{jt} + T_{jt} - ES_{jt}$ in Equation (5.1). Lastly, $\mathbf{J}_{ft}$ denotes the set of car models produced by manufacturer f in year t.

The first-order condition (FOC) of the profit maximization problem is as follows:

$$\frac{\partial \pi_{ft}}{\partial p_{jt}} = q_{jt}\left(\mathbf{p}_t^e\right) + (1+\rho_{jt}) \sum_{l \in \mathbf{J}_{ft}} (p_{lt} - mc_{lt}) \frac{\partial q_{lt}}{\partial p_{jt}^e} = 0, \ \forall j \in \mathbf{J}_{ft} \tag{6.2}$$

By stacking the FOCs across all products, we obtain the equilibrium conditions for year t in the following matrix notation:

$$\tilde{\mathbf{q}}_t(\mathbf{p}_t^e) - D_t(\mathbf{p}_t^e)(\mathbf{p}_t - \mathbf{mc}_t) = \mathbf{0}, \tag{6.3}$$

where $\tilde{\mathbf{q}}_t = (\frac{q_{1,t}}{1+\rho_{1t}}, \cdots, \frac{q_{J_t,t}}{1+\rho_{J_t,t}})'$, $\mathbf{p}_t = (p_{1,t}, \cdots, p_{J_t,t})'$, and $\mathbf{mc}_t = (mc_{1,t}, \cdots, mc_{J_t,t})'$. The matrix D_t is a $J_t \times J_t$ matrix defined as $D_t(\mathbf{p}_t^e) = \Omega_t \odot S(\mathbf{p}_t^e)$. Here the operator $\odot$ denotes the element-by-element multiplication of matrices. Ω_t, meanwhile, denotes the ownership structure of car models sold in market t. More specifically, the (i, j) element of the matrix Ω_t takes a value of 1 if product i and j are sold by the same manufacturer, and 0 otherwise. Lastly, the (i, j) element of matrix $S(\mathbf{p}_t^e)$ is defined as $-\frac{\partial q_{jt}(\mathbf{p}_t^e)}{\partial p_{it}^e}$.

To use this supply model for simulations, we must first estimate the model primitives, namely the demand function and marginal costs. The demand function is estimated in Section 5. We use the equilibrium conditions to estimate the vector of marginal cost $\mathbf{mc}_t$. Specifically, given that the matrix $S(\mathbf{p}_t^e)$ can be calculated from the demand estimates, we can invert Equation (6.3) to back out the marginal costs $\mathbf{mc}_t$.

Given the estimated model primitives, we conduct a simulation analysis by numerically solving Equation (6.3) for a vector of equilibrium prices. To do so, we use the algorithm proposed by Morrow and Skerlos (2011).[37]

6.2 Simulation 1: Pass-through Analysis of Feebate Policy

In this subsection, we conduct a pass-through analysis of eco-car subsidies, simulating the market equilibrium when the subsidy for eligible automobile models is removed (i.e., $ES_{jt} = 0$ for all j in Equation (5.1)). We then calculate how much of the subsidy can be attributed to consumers and producers. Note that our simulation aims to highlight the practical value of our demand framework in the pass-through analysis rather than fully evaluating the Japanese feebate policy.[38]

First, we report the pass-through rate in Table 8. We define the pass-through rate (PTR_{jt}) as the ratio of changes in effective price that consumers face to the amount of ES:

$$PTR_{jt} = \frac{p_{jt}^{e'} - p_{jt}^{e}}{ES_{jt}},$$

where $p_{jt}^{e'}$ indicates the simulated price in the counterfactual case without ES. The table shows the results based on three different demand specifications: (1) semiparametric model, (2) parametric income effect, and (3) linear logit. We find that the average pass-through rate under our semiparametric model and parametric random coefficient model are similar (1.194 and 1.196), while the rate implied by a linear logit model is 0.991. In addition, the pass-through rate under a linear logit model is bounded by 1. This result is consistent with Weyl and Fabinger (2013)'s notion that log-concave demand always predicts an incomplete pass-through (i.e., the pass-through rate is below 1).

[37] The algorithm of Morrow and Skerlos (2011) is used in pyBLP package provided by Conlon and Gortmaker (2020)

[38] We abstract away several institutional features in our analysis. First, the actual policy began in the middle of the year (i.e., in April), but our data is annual, so we cannot fully account for this point. Thus, we assume that the first phase of the ES policy was implemented from April 2009 to September 2010, and that the second ran from December 2011 to January 2013 (see Konishi and Zhao (2017), who analyzed the policy using quarterly data). Second, in the policy's first phase, consumers could apply for the ES program with a higher amount of subsidy if they scrapped an existing vehicle older than 13 years. To analyze the scrap subsidy, we must incorporate consumer heterogeneity for the age of the owned vehicle. For an evaluation of the feebate policy with full consideration of the scrap subsidy, see Kitano (2022).

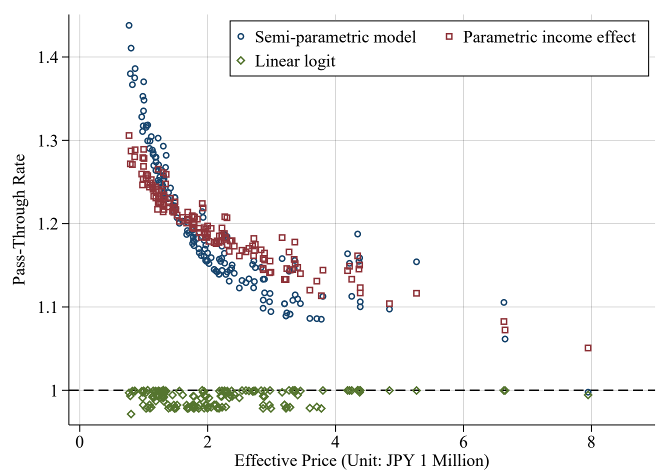
Table 8: Price Change Due to Ecocar Subsidy

	Mean	SD	p25	Median	p75
A: PTR_{jt}					
Semi-parametric model	1.194	0.080	1.140	1.174	1.252
Parametric income effect	1.196	0.046	1.166	1.194	1.229
Linear logit	0.991	0.009	0.980	0.993	0.999
B: Percentage Change in Effective Price					
Semi-parametric model	-5.968%	2.434%	3.997%	5.701%	7.532%
Parametric income effect	-5.940%	2.287%	4.068%	5.832%	7.232%
Linear logit	-4.928%	1,788%	3.500%	4.852%	5.932%

Note: $PTR_{jt} = (p^{e'}_{jt} - p^{e}_{jt})/ES_{jt}$. Percentage change in effective price due to ecocar subsidy is defined as $100 * (p^{e'}_{jt} - p^{e}_{jt})/p^{e'}_{jt}$. p25 and p75 represent the 25th and 75th percentiles, respectively.

To examine the difference between our semiparametric model and parametric random coefficient specification, Figure 10 shows the relationship between the pass-through rate and the effective price for three different demand specifications. While the average pass-through rates across baseline and parametric models are similar, the baseline specification shows more significant heterogeneity in the pass-through rate. Specifically, it predicts a higher pass-through rate for cheaper products and a lower rate for more expensive products. Such heterogeneity in our semiparametric model is consistent with the pattern in the curvature estimates discussed in Figure 9.

Figure 10: Pass-Through Rate of the Eco-Car Subsidy



Note: The car models whose effective price is less than JPY 8 million are plotted.

We now quantify the policy impact on social welfare. We calculate the changes in producer surplus (PS_t), tax revenues (TR_t), and consumer surplus measured by compensation variation

(CV).[39] For a simple logit and a parametric random coefficient model, we use Small and Rosen (1981) to calculate the consumer surplus.[40] For the semiparametric specification, we compute the compensating variation following Dagsvik and Karlström (2005).[41] Table 9 reports the results under three demand specifications. Regardless of the specification, the ES policy improves the total welfare. In particular, the increase in consumer surplus exceeds the fiscal expenditure of the subsidy. This improvement is likely due to the subsidy mitigating the pre-existing distortion caused by market power (Buchanan 1969; Fowlie et al. 2016).

Table 9: Welfare Impact of Ecocar Subsidy

	2009	2010	2012	Average
(i) Semi-parametric model				
Consumer Surplus	302.2	406.4	571.7	426.8
Profit	97.9	129.5	174.5	134.0
Tax Revenue	-143.4	-197.4	-274.4	-205.1
Total Welfare	256.7	338.6	471.8	355.7
(ii) Parametric income effect				
Consumer Surplus	179.6	250.2	344.1	258.0
Profit	101.8	136.6	187.1	141.8
Tax Revenue	-136.7	-188.3	-261.2	-195.4
Total Welfare	144.7	198.5	269.9	204.4
(iii) Linear logit				
Consumer Surplus	153.0	211.5	293.7	219.4
Profit	138.9	191.7	263.7	198.1
Tax Revenue	-144.2	-198.1	-277.5	-206.6
Total Welfare	147.7	205.1	279.9	210.9

Note: The unit is JPY 1 Billion. The average of 2009, 2010, and 2012 is reported in the final column.

Table 10 reports the distribution of compensating valuation. The linear logit model in Panel (iii) removes consumer heterogeneity, and thus predicts the uniform impacts on consumer surplus. However, the other two models incorporate the consumer heterogeneity into the model and thus predict the heterogenous impacts of the policy change. Overall, we find a large standard deviation in Panels (i) and (ii), which implies significant heterogeneity in policy effects among consumers. The CV implied by the simple logit model in Panel (iii) is between the median and the 75th percentile

[39] The producer surplus is $PS_t = \sum_{f \in F} \sum_{j \in \mathbf{J}_{ft}} (p_{jt} - mc_{jt}) q_{jt}(\mathbf{p_t^e})$ and the tax revenue is given by $TR_t = \sum_{f \in F} \sum_{j \in \mathbf{J}_{ft}} (p_{jt}\rho_{jt} + T_{jt} - ES_{jt}) q_{jt}(\mathbf{p_t^e})$. Note that F denotes the set of automobile manufacturers.

[40] For a parametric random coefficient model, we use the log-sum formula of Small and Rosen (1981) to each consumer with different income level y_{it}. We then aggregate individual consumer surplus to calculate the aggregate consumer surplus.

[41] See Appendix H for further details on how CV is calculated. We aggregate the individual CV across different consumers in Table 9.

of the distribution under our model with a nonparametric income effect.

Table 10: The Distribution of CV by Year (Ecocar Subsidy)

	Mean	SD	p10	p25	Median	p75	p90
(i) Semi-parametric model							
2009	5,715	11,705	0	0	6	3,280	26,511
2010	7,616	15,822	0	0	14	4,202	34,570
2012	10,554	20,189	0	0	28	8,024	50,137
(ii) Parametric income effect							
2009	3,397	7,302	0	8	229	2,525	11,375
2010	4,689	10,390	0	13	301	3,280	15,390
2012	6,351	12,442	1	27	605	5,667	22,124
(iii) Linear logit							
2009	2,893	NA	NA	NA	NA	NA	NA
2010	3,963	NA	NA	NA	NA	NA	NA
2012	5,423	NA	NA	NA	NA	NA	NA

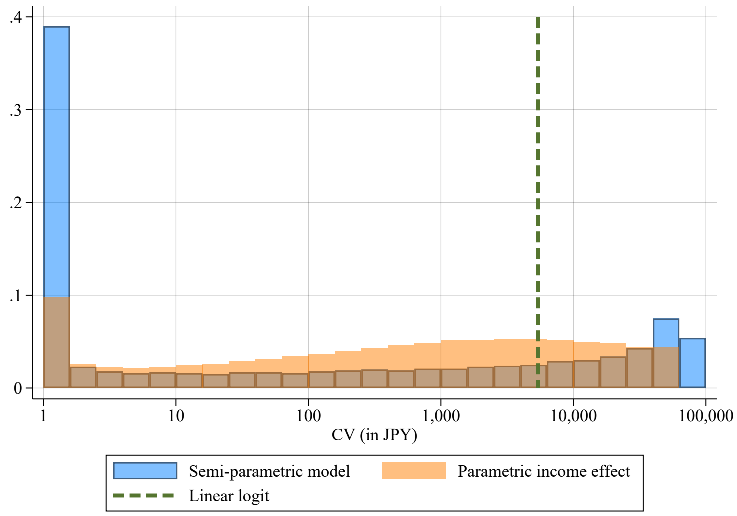
Note: The unit is JPY. p10, p25, p75, and p90 represent the 10th, 25th, 75th, and 90th percentile points respectively. Under a linear Logit specification, there is no heterogeneity in CV.

To highlight the difference between Panels (i) and (ii), Figure 11 shows the CV distribution in a parametric income effect and semiparametric models. The distribution of CV under a parametric income effect specification is right-skewed, but bimodal under a semiparametric model. Indeed, the semiparametric model implies that the policy has heterogenous implications on consumers. This result suggests the importance of flexibly incorporating the income effect when analyzing the distributional effects of the policy on consumers.

6.3 Simulation 2: Merger Simulation

We conduct a simulation analysis of a hypothetical merger between Toyota and Honda. As in the pass-through analysis in Section 6.2, we compare the price effects of a merger under three demand specifications. To evaluate a merger's impact, we solve the counterfactual outcome by solving the equilibrium conditions under an alternative ownership structure. Specifically, we set the ownership matrix Ω_t such that the car models produced by Toyota and Honda are owned by the same firm. The hypothetically merged firm chooses prices to maximize the joint profit. Note that we use the estimated marginal costs and do not consider potential efficiency gains from the merger. Here, we focus on the anti-competitive effects, which are of primary interest in the antitrust practice and are determined by the demand structure.

Figure 11: Distribution of CV in 2012



Note: The common logarithm scale is applied to the x-axis. To avoid missing observations, we add 1 to the estimated CVs.

Table 11 presents the simulation results. In Panel (D), the price effects of a merger are the smallest in a linear logit model. While the product prices increase by 2.7% for Toyota and 7.3% for Honda under a semiparametric specification, a simple logit demand predicts increases of 0.6% for Toyota and 1.4% for Honda.

We now investigate the underlying mechanism that drives the difference in the predicted merger effects. As discussed in Farrell and Shapiro (2010), the price effects of a merger can be considered to be, in the first order, the pass-through of the increase in the opportunity costs. After the merger, the merged firm takes into account the opportunity cost of losing the partner's profits. This measure is referred to as the upward pricing pressure (UPP), defined in greater detail below. The price effect of a merger is determined by the extent to which the increase in opportunity cost is reflected in the final price. Therefore, the key to understanding the merger effect is the pass-through rate and the UPP implied from each demand specification.

As seen in the pass-through analysis, the estimated pass-through rate is much higher in the semiparametric and parametric random coefficient models than in the simple logit model. This is largely due to the difference in the estimated curvature of the demand curve.

To investigate the difference in the UPP and how it relates to the differential merger effect,

Table 11: Descriptive Statistics of Merger Simulation

	Mean	SD	p25	Median	p75
A: Observed Effective Price (Unit: JPY 1 million)					
(1) Toyota	2.76	2.01	1.76	2.18	2.98
(2) Honda	2.51	1.27	1.46	2.33	2.98
B: Effective Price Change in % (Semi-parametric)					
(1) Toyota	2.68%	0.60%	2.38%	2.74%	3.08%
(2) Honda	7.30%	0.86%	6.78%	7.46%	7.93%
C: Effective Price Change in % (Parametric income effect)					
(1) Toyota	1.87%	0.33%	1.74%	1.94%	2.10%
(2) Honda	5.32%	0.51%	4.99%	5.51%	5.71%
D: Effective Price Change in % (Linear Logit)					
(1) Toyota	0.59%	0.25%	0.42%	0.57%	0.74%
(2) Honda	1.37%	0.62%	0.91%	1.23%	1.82%

Note: p25 and p75 represent the 25th and 75th percentiles.

we compute the UPP following Farrell and Shapiro (2010) and Miller et al. (2016). For ease of explanation, let us consider a merger between two firms, A and B. Firm A's products are denoted by $\{1, \ldots, J_A\}$, while those of firm B are denoted by $\{J_A + 1, \ldots, J_A + J_B\}$. Then, the UPP for firm A is defined as follows:[42]

$$\begin{bmatrix} UPP_1 \\ \vdots \\ UPP_{J_A} \end{bmatrix} = - \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_1}{\partial p_{J_A}} & \cdots & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial q_{J_A+1}}{\partial p_1} & \cdots & \frac{\partial q_{J_A+J_B}}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_{J_A+1}}{\partial p_{J_A}} & \cdots & \frac{\partial q_{J_A+J_B}}{\partial p_{J_A}} \end{bmatrix} \begin{bmatrix} p_{J_A+1} - mc_{J_A+1} \\ \vdots \\ p_{J_A+J_B} - mc_{J_A+J_B} \end{bmatrix}. \tag{6.4}$$

The first two terms in the right-hand side is a matrix of diversion ratios from firm A to firm B. The product of the diversion ratios and the markup of firm B captures an opportunity cost created by the merger. We compute the UPP evaluated at the price without the merger.

Table 12 reports the descriptive statistics of the UPP across different demand specifications. The linear logit model implies the lowest UPP among three specifications. Combined with the lower pass-through rate, the linear logit model predicts the smallest merger effects. Comparing the semiparametric and parametric BLP models, the UPP under the former is greater than that under the latter. While both models are, on average, similar in the pass-through rate (as discussed in Section 6.2), the difference in the UPP leads to the differential merger effects.

[42] For the full derivation of the above equation, see Appendix F.

Table 12: Descriptive Statistics of UPP

	Mean	SD	p25	Median	p75
Semi-parametric model					
(1) Toyota	4.96	1.32	3.99	4.77	5.89
(2) Honda	13.05	5.32	8.30	12.71	16.40
Parametric income effect					
(1) Toyota	3.48	1.14	2.67	3.24	4.16
(2) Honda	9.82	4.44	6.01	9.43	12.55
Linear logit					
(1) Toyota	1.21	0.15	1.15	1.18	1.38
(2) Honda	2.61	0.28	2.52	2.72	2.88

Note: p25 and p75 represent the 25th and 75th percentiles. Unit is JPY 10 thousand.

Lastly, Table 13 shows the welfare effects of the merger. Our semiparametric model predicts that the merger will result in a larger loss of consumer surplus and total welfare, which reflects the larger price effects in the semiparametric specification.

Table 13: Welfare Impact of Merger

	Semi-parametric model	Parametric income effect	Linear logit
Consumer Surplus	-240.1	-108.9	-33.8
Profit	35.9	21.9	1.6
Tax Revenue	-11.8	-12.9	-2.2
Total Welfare	-216.0	-99.9	-34.4

Note: The average welfare impact from 2006 to 2013 is reported, as measured in billion JPY.

7 Discussion and Robustness in Empirical Application

This section discusses the robustness of our empirical application. We first examine the use of rental (rather than regular) prices of automobiles in our empirical application in Section 7.1. We then discuss the implications of omitting the budget constraint in our demand model in Section 7.2.

7.1 Rental Price

Contrary to many discrete choice demand models, our model explicitly considers the presence of budget constraints. In our empirical application, we use the annual household income and the effective automobile prices after considering tax and subsidy. However, given that people tend to finance vehicle purchases either with auto loans or savings, our specification of budget constraints may not reflect the actual budget constraint faced by consumers.

To mitigate this concern, we construct the rental price of automobiles and use it in our estimation and simulation analysis. Some previous papers (e.g., Bento et al. (2009)) have used rental (rather than purchase) price in their demand models. We follow Abe (2023) to construct the rental price of automobiles, the details of which are presented in Appendix G. After constructing the rental price, we re-run the estimation and simulation analysis to assess the robustness against the choice of price measure.

Table 14 shows the comparison of the results of the pass-through analysis in Section 6.2. Regardless of whether using the original definition of price (i.e., effective price) or the rental price, the estimated pass-through rates are quite similar. These results suggest that our demand model is robust to the choice of price measure (i.e., the purchase price or the rental price).

Table 14: Price Change Due to Ecocar Subsidy (Original Model versus Rental Price)

	Mean	SD	p25	Median	p75
A: PTR_{jt}					
Semi-parametric model (Original)	1.194	0.080	1.140	1.174	1.252
Semi-parametric model (Rental Price)	1.194	0.074	1.141	1.181	1.239
B: Percentage Change in Effective Price					
Semi-parametric model (Original)	−5.97%	2.43%	−4.00%	−5.70%	−7.53%
Semi-parametric model (Rental Price)	−5.97%	2.47%	−3.97%	−5.72%	−7.30%

Note: p25 and p75 represent the 25th and 75th percentiles. See Appendix G for how to derive PTR in the model using rental price.

7.2 Role of Budget Constraint

This subsection addresses the implications of a budget constraint for the pass-through analysis. In the empirical application, we approximate the income effect term $f(y-p)$ by the inverse of Bernstein polynomial function, given by $f(y-p) = \frac{-1}{\sum_{k=1}^{K} \pi_k b_k^K(y-p)}$ where $\lim_{x\to 0} f(x) = -\infty$. As discussed in Section 5.3, this specification yields the continuous demand function, which is vital for supply-side analysis. However, one might view this as an unnecessary restriction that could affect the implications of empirical analysis.[43] To address this concern, we conduct the pass-through analysis under an alternative specification of the demand model where the budget constraint is omitted.

We consider the following demand model without the budget constraint:

$$s_{jt} = \int s_{ijt}(y_{it}) dG_t(y_{it}) \tag{7.1}$$

where

$$s_{ijt}(y_{it}) = \frac{\exp\left(f(y_{it} - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)}{\exp(f(y_{it})) + \sum_{k=1}^{J_t} \exp\left(f(y_{it} - p_{kt}) + \beta' X_{kt} + \xi_{jt}\right)}. \tag{7.2}$$

In contrast to the baseline specification of Equation (2.10), the individual choice probability does not incorporate the budget constraint $\mathbf{1}\{y_{it} > p_{jt}\}$, implying that the individual choice probability s_{ijt} can be non-zero even when $p_{jt} < y_{it}$ (i.e., the budget constraint for a consumer with income level y_{it} is violated) and thus be continuous at $p_{jt} = y_{it}$. Regarding the income effect term $f(y-p)$, we adopt the approximation by a Bernstein polynomial $f(y_{it} - p_{jt}) = \sum_{k=0}^{K} \pi_k b_k^K (y_{it} - p_{jt}) \equiv \psi^K (y_{it} - p_{jt})'\Pi$. We set $K = 4$ as in the baseline specification. We also normalize the support of $y_{it} - p_{jt}$ to $[0, 1]$.[44]

The advantage of this specification is that the demand model becomes a continuous function for a price, as it does not include the budget constraint (see Appendix E for details). Therefore, in contrast to the baseline specification, the inverse of the Bernstein polynomial is unnecessary for maintaining the continuity of demand function. However, the disadvantage is that the model may not align with the underlying utility maximization behavior, as consumers might violate the budget constraint. Therefore, under this specification, conducting a coherent welfare analysis (i.e., calculating consumer welfare) becomes challenging.

Table 15 presents the simulated pass-through rates under two specifications: the baseline model and the model without a budget constraint. Although the baseline model predicts, on average, a

[43] We conduct a Monte Carlo experiment using the inverse of the Bernstein polynomial rather than the regular Bernstein polynomial as we did in Section 4. Since the inverse is restricted to satisfy $\lim_{y-p\to 0} f(y-p) = -\infty$, we could not recover the true functional form with normalization of $f(y-p) = 0$ (see DGP1-3 introduced in Section 4.) The detailed result is available upon request.

[44] Footnote 15 explains the normalization of the support when we consider the presence of budget constraints. Without the budget constraint where $y_{it} - p_{jt}$ can take a negative value as an argument of the function $f(\cdot)$, we construct z_{min} by $z_{min} = \min_{i,j,t}\{y_{it} - p_{jt}\}$ and use it to construct the standardized variable.

slightly higher pass-through rate, the overall pattern remains quite similar. Notably, regardless of the specification chosen, the model consistently predicts the over-pass-through of the subsidy.

Table 15: Price Change Due to Ecocar Subsidy (Original Model versus No Budget Constraint)

	Mean	SD	p25	Median	p75
A: PTR_{jt}					
Semi-parametric model (Original)	1.194	0.080	1.140	1.174	1.252
Semi-parametric model (No Budget Consraint)	1.105	0.081	1.041	1.075	1.157
B: Percentage Change in Effective Price					
Semi-parametric model (Original)	−5.97%	2.43%	−4.00%	−5.70%	−7.53%
Semi-parametric model (No Budget Constraint)	−5.36%	1.69%	−4.05%	−5.17%	−6.15%

Note: See note in Table 8.

We conclude the discussion by highlighting the trade-offs associated with the choice of approximation methods. In the baseline specification of empirical analysis, we choose to impose the budget constraint, thus adopting the inverse of the Bernstein polynomial. We do this to conduct a comprehensive demand and supply analysis, including counterfactual simulations and the measurement of consumer welfare. On the other hand, if one's interest lies in recovering the demand function with minimal assumptions, using regular Bernstein polynomials without imposing the budget constraint might be a preferable approach. However, we should be aware that such a model may not align with the underlying principle of utility maximization, and it does not allow us to measure consumer welfare. Additionally, omitting the budget constraint may lead to biased demand estimates (see, e.g., Pesendorfer et al. (2023)).

8 Conclusion

This paper proposes a new empirical framework for a differentiated product demand model with a nonparametric income effect. The proposed model is a semiparametric model with endogeneity. We estimate the model by combining the NFP algorithm proposed by BLP and a sieve approximation with shape restriction. Our Monte Carlo simulations suggest significant gains in estimating the nonparametric term of the income effect by incorporating the shape restriction. We also apply our empirical framework to Japanese automobile data, demonstrating the importance of a flexible income effect specification. In our application of the pass-through analysis and the merger simulation, our demand model offers qualitatively and quantitatively different results than the parametric models.

Appendix for Online Publication

Demand Estimation with Flexible Income Effect:

An Application to Pass-through and Merger Analysis

Shuhei Kaneko and Yuta Toyama

A Additional Tables and Figures

Table A1: Regression of Automobile Prices

	(1)	(2)	(3)
Eco-car subsidy	-1.7724		-1.1479
	(0.1492)		(0.1641)
Specific Tax		3.2525	2.2680
		(0.2704)	(0.3011)
HP/WT	10.1043	9.2747	9.4539
	(1.2233)	(1.2221)	(1.1976)
MPG	-0.0038	-0.0007	-0.0004
	(0.0025)	(0.0025)	(0.0024)
Car Size	0.9052	1.0271	1.0064
	(0.2566)	(0.2527)	(0.2467)
AT/CVT	0.1177	0.0774	0.0892
	(0.0335)	(0.0181)	(0.0210)
Hybrid dummy	0.6985	0.8521	0.7982
	(0.1843)	(0.1732)	(0.1785)
Constant	-5.0243	-6.1898	-5.9895
	(1.8419)	(1.8100)	(1.7712)
FE	Product and Year	Product and Year	Product and Year
Observations	1,302	1,302	1,302

Notes: We run the regression $p^e_{jt} = \beta X_{jt} + \gamma ES_{jt} + \gamma Tax_{jt} + u_{jt}$, where p^e_{jt} is the effective price (including tax), X_{jt} is a vector of product characteristics, ES_{jt} is the amount of eco-car subsidy, and Tax_{jt} is the amount of auto-related taxes excluding acquisition tax. We also add the product and the year-fixed effects.

Table A2: Estimation Results of Parametric Income Model

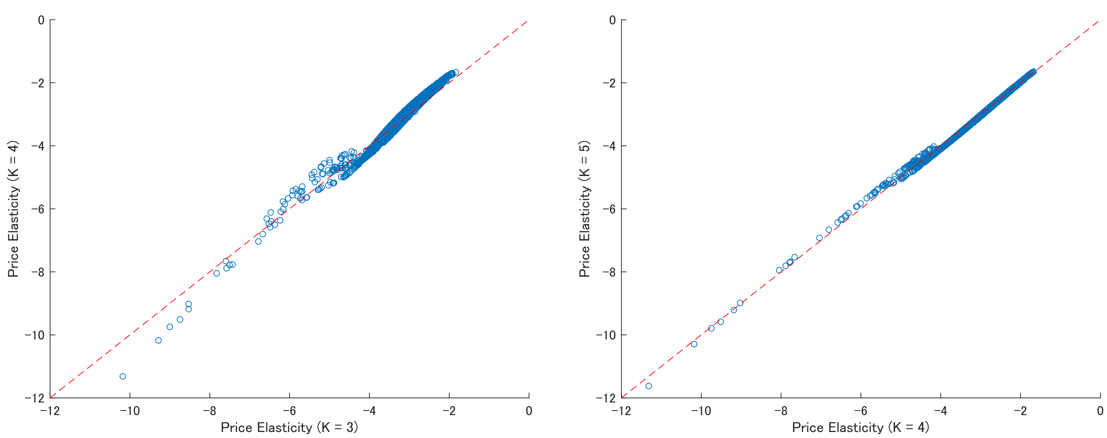
	Point Est.	Standard Error
Constant	-27.507	1.194
HPWT	26.820	4.907
MPG	0.070	0.009
Size	2.745	0.228
AT/CVT	0.362	0.399
$\frac{p^e_{jt}}{y_i}$	22.176	4.678
Observations	1,322	

Notes: The model includes year fixed effects, firm fixed effects, minicar dummy and hybrid car dummy.

Figure A1: Comparison of Price Elasticities across Different Choice of the order of Bernstein Polynomial K

Panel A: $K = 3$ and $K = 4$

Panel B: $K = 4$ and $K = 5$



B Income Distribution

Income distribution is obtained from *the Comprehensive Survey of Living Conditions (CSLC)*, which is conducted annually in Japan by the Ministry of Health, Labor, and Welfare (MHLW). Specifically, we used the summary of CSLC annual data circulated by MHLW in which the median and the average income of the surveyed population are reported. In our analysis, we assume that the annual income follows a log-normal distribution ($= LN(\mu_t, \sigma_t^2)$). The parameters are calibrated using the property that $\mathbb{E}(y) = \exp(\mu + \sigma^2/2)$ and $Median(y) = \exp(\mu)$ if y follows $LN(\mu, \sigma^2)$. Table A3 shows the nominal and real (i.e., deflated by 2015 CPI) average and median income and the parameters of income distribution from 2006 to 2013.

Table A3: Descriptive Statistics of Annual Income Data from CSLC

Year	Average[(a)]	Median[(a)]	Average (Deflated)	Median (Deflated)	μ_t[(b)]	σ_t[(b)]
2006	566.8	451	583.1276	463.9918	6.1399	0.6761
2007	556.2	448	572.2222	460.9053	6.1332	0.6578
2008	547.5	427	555.2738	433.0629	6.0709	0.7051
2009	549.6	438	565.4321	450.6173	6.1106	0.6738
2010	538.0	427	557.5130	442.4870	6.0924	0.6798
2011	548.2	432	569.2627	448.5981	6.1061	0.6902
2012	537.2	432	558.4200	449.0644	6.1072	0.6602
2013	528.9	415	547.5155	429.6066	6.0629	0.6964

Notes: The unit used in columns 2–5 is JPY 10 thousands. (a): Average and median income are sourced from the summary of CSLC circulated by MHLW. (b) The parameters of the log-normal distribution is calculated based on deflated average/median income.

C Tax Policy in Japanese Automobile Market

This appendix explains the tax system in the Japanese automobile market. Under the Japanese vehicle tax system, consumers must pay three types of car tax: (1) acquisition tax, (2) weight tax, and (3) automobile tax. The acquisition tax is an ad valorem tax, while the other two are specific taxes that depend on the weight and engine displacement of the car model.

First, the automobile acquisition tax is an ad valorem tax collected by each prefecture, which charges 5% (3%) of the purchase price before March 2014 (after April 2014). Note that the automobile acquisition tax was abolished in October 2019, with a new taxation system called the environmental performance discount (*Kankyo-Seino-Wari* in Japanese) being rolled out in October 2019. Under this new system, at most 3% of the purchase price is imposed depending on the automobile's fuel efficiency.

Second, the rate of the automobile weight tax was JPY $12,600$ per 0.5 tons of vehicle weight before April 2010, JPY 10,000 per ton from April 2010 to April 2012, and JPY 8,200 per ton after May 2012.

Third, the automobile tax is an additional tax collected by each prefecture. In recent years, the size of the automobile tax has been modified several times. For instance, the automobile tax on minicars was hiked from JPY $7,200$ to JPY $10,800$, while the range of the automobile tax on normal cars was raised from JPY $29,500$–$111,000$ to JPY $25,000$–$110,000$ in October 2019.

Furthermore, in 2009, a tax reduction was introduced for car models that satisfy criteria based on fuel efficiency and emissions standards. This tax reduction scheme is called the eco-car tax reduction (hereafter ETR). The changes in the tax reduction rates and the criteria of the ETR program are described in Table A4. The eligibility for the tax reduction was revised in 2012, 2014, and 2015. For instance, from 2009 to 2011, the acquisition tax on new cars that met the 2010 fuel efficiency standards by 15% or better and received a four-star rating for the emission standard in 2005 was cut by 50%, while the automobile tax on these cars was reduced by 25%.

Table A4: Eligibility for Tax Reduction Under the ETR program

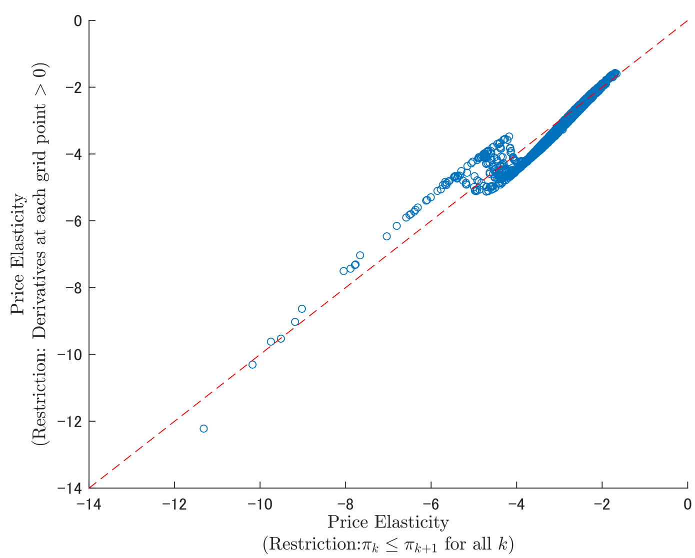
	Acquisition tax	Weight tax	Automobile tax	
			Normal	Minicar
(1) 2009−2011				
EV, FCV, Plug-in Hybrid, etc.	Exempted	Exempted	50% cut	No exemption
125% or above 2010 standard	75% cut	75% cut	50% cut	
115% or above 2010 standard	50% cut	50% cut	25% cut	
(2) 2012−2013				
EV, FCV, Plug-in Hybrid, etc.	Exempted	Exempted	50% cut	
120% or above 2015 standard	Exempted	Exempted	50% cut	No exemption
110% or above 2015 standard	75% cut	75% cut	50% cut	
100% or above 2015 standard	50% cut	50% cut	25% cut	
(3) 2014				
EV, FCV, Plug-in Hybrid, etc.	Exempted	Exempted	75% cut	
120% or above 2015 standard	Exempted	Exempted	75% cut	No exemption
110% or above 2015 standard	80% cut	75% cut	50% cut	
100% or above 2015 standard	60% cut	50% cut	50% cut	
(4) 2015−2016				
EV, FCV, Plug-in Hybrid, etc.	Exempted	Exempted	75% cut	75% cut
120% or above 2020 standard	Exempted	Exempted	75% cut	50% cut
110% or above 2020 standard	80% cut	75% cut	75% cut	25% cut
100% of above 2020 standard	60% cut	50% cut	75% cut	25% cut
110% above 2015 standard	40% cut	25% cut	50% cut	No exemption
105% above 2015 standard	20% cut	25% cut	50% cut	

Notes: For all tax reductions, automobiles must receive a four-star rating for the emission standards in 2005. ETR: Eco-car Tax Reduction, ES: Eco-Car Subsidy, EV: Electronic Vehicle, FCV: Fuel-Cell Vehicle.

D Alternative Way to Impose Shape Restriction

To impose the shape restriction on the income effect function $f(x)$ in a sieve approximation, we impose the condition that $\pi_k \leq \pi_{k+1}$ for all k in the Bernstein polynomial. This is a sufficient, rather necessary, condition for the function $f(x)$ to be weakly-increasing. An alternative approach would be to impose the restriction on a grid of points on the support. We investigate this approach and compare the result with the baseline approach (i.e., imposing $\pi_k \leq \pi_{k+1}\ \forall k$) in our empirical application. Specifically, we consider an alternative constraint that the derivative of the Bernstein polynomial is positive at 999 points of an even grid (i.e., 0.1 percentage tile to 99.9 percentage tile) in our optimization of the sieve GMM. Figure A2 shows the estimated own-price elasticity from these two approaches. Overall, both approaches produce quite similar values of the own-price elasticity. In terms of computation speed, the baseline approach is three times faster than the alternative approach. Based on this finding, we suggest that directly imposing the constraint on sieve coefficient π_k is computationally superior without compromising the generality.

Figure A2: Comparison of Estimation Results of Different Shape Restrictions



Note: Estimated own-price elasticity under two possible ways to impose shape restriction on $f(y-p)$ is plotted: our baseline approach (i.e., imposing increasing sieve coefficients $\pi_k < \pi_{k+1}$ for all k) on the x-axis and alternative approach (imposing positive derivative at 999 even grid points) on the y-axis.

E Numerical challenge

In our empirical analysis in Section 5, we impose the additional restriction of $\lim_{x\to 0} f(x) = -\infty$ on the income effect function. This appendix provides a detailed explanation of this restriction by employing a numerical example.

In our demand model with budget constraint, the aggregate demand function is given by

$$s_{jt} = \int s_{ijt}(y_{it}) dG_t(y_{it})$$

where

$$s_{ijt}(y_{it}) = \frac{\mathbf{1}\{y_{it} \geq p_{jt}\} \cdot \exp\left(f(y_{it} - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)}{\exp(f(y_{it})) + \sum_{k=1}^{J_t} \mathbf{1}\{y_{it} \geq p_{kt}\} \cdot \exp\left(f(y_{it} - p_{kt}) + \beta' X_{kt} + \xi_{jt}\right)}.$$

Since we are not able to obtain the analytical formula for the above integral, we rely on a Monte Carlo simulation:

$$\begin{aligned} s_{jt} &= \frac{1}{R} \sum_{r=1}^{R} s_{ijt}(y_r) && \text{(E.1)} \\ &= \frac{1}{R} \sum_{r=1}^{R} \frac{\mathbf{1}\{y_r \geq p_{jt}\} \cdot \exp\left(f(y_r - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)}{\exp(f(y_r)) + \sum_{k=1}^{J_t} \mathbf{1}\{y_r \geq p_{kt}\} \cdot \exp\left(f(y_r - p_{kt}) + \beta' X_{kt} + \xi_{jt}\right)} && \text{(E.2)} \end{aligned}$$

where $\{y_r\}_{r=1}^{R}$ is the set of simulated income randomly drawn from the income distribution $G_t(y)$.

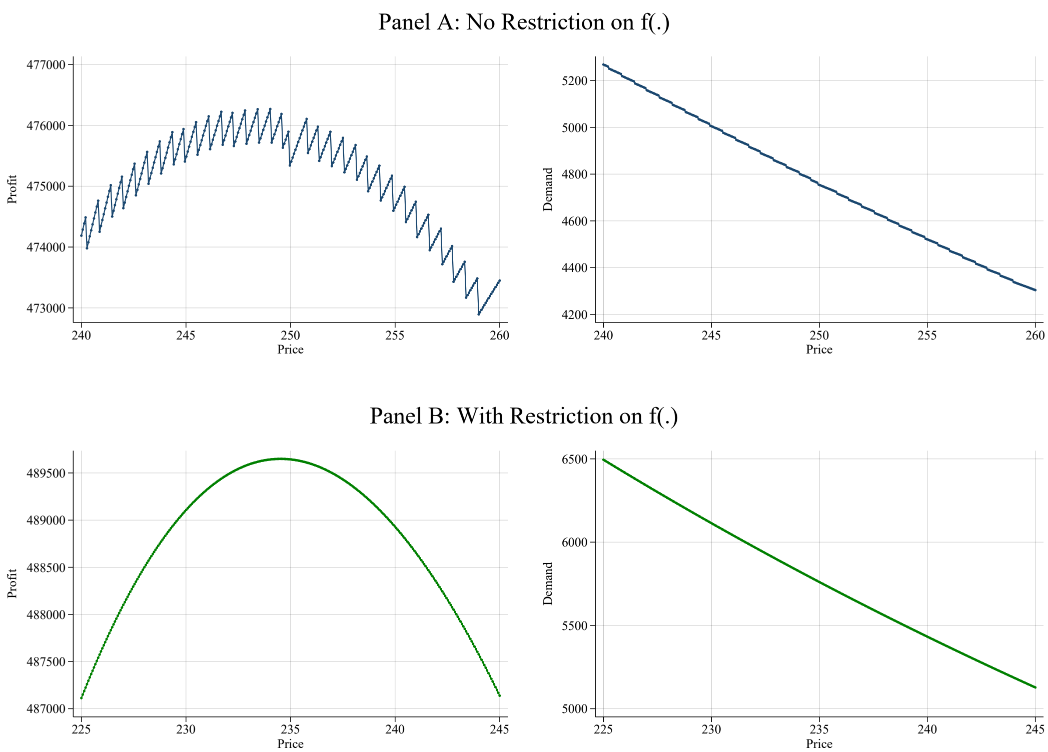
The key issue is that due to the presence of indicator function $\mathbf{1}\{y_r \geq p_{jt}\}$ in Monte Carlo integral, the market-level demand s_{jt} is discontinuous when $p_{jt} = y_r$. We can see this point graphically in Figure A3. We plot the demand curve and the profit function of a particular product with respect to its own price, holding other prices fixed. In the upper-panel of Figure A3, we set the income effect function as $f(y - p) = 30(y - p)$ implying that $f(0) = 0$ and thus the exponential function of the numerator $\exp\left(f(y_r - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)$ is a positive number when $p_{jt} = y_r$. As the upper-left panel shows, the demand curve is discontinuous at the points where the price is equal to the level of simulated income. Such discontinuities in demand affect the shape of the profit function, as shown in the upper-right panel. The profit function is no longer a smooth function with respect to prices, making the numerical analysis of finding a profit-maximizing price difficult.

To avoid this issue, we decided to impose the restriction that $\lim_{x\to 0} f(x) = -\infty$. Under this restriction, the exponential function of the numerator $\exp\left(f(y_r - p_{jt}) + \beta' X_{jt} + \xi_{jt}\right)$ goes to 0 as $p_{jt} \to y_r$. We graphically illustrate this point by considering another specification for the income effect function

$$f(y - p) = -\frac{1}{0.2(y - p)},$$

which is similar to the one we adopt in an empirical analysis of the main body. As the lower panel shows, both the demand and profit function is smooth with respect to its prices. Therefore, imposing such additional restrictions is crucial to using our framework for a numerical simulation of solving the pricing game.

Figure A3: Illustration of Numerical Challenge



F Derivation of the Upward Pricing Pressure

This appendix shows the derivation of the upward pricing pressure (UPP). As in the main body, consider a merger between two firms A and B. Firm A's products are denoted by $\{1, \ldots, J_A\}$, while those of firm B are denoted by $\{J_A + 1, \ldots, J_A + J_B\}$.

Before the merger, the system of FOCs for firm A is given by

$$\begin{bmatrix} \tilde{q}_1(\mathbf{p}_t^e) \\ \vdots \\ \tilde{q}_{J_A}(\mathbf{p}_t^e) \end{bmatrix} + \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ & \ddots & \\ \frac{\partial q_1}{\partial p_{J_A}} & & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix} \begin{bmatrix} p_1 - mc_1 \\ \vdots \\ p_{J_A} - mc_{J_A} \end{bmatrix} = 0, \tag{F.1}$$

or, equivalently,

$$\begin{bmatrix} p_1 - mc_1 \\ \vdots \\ p_{J_A} - mc_{J_A} \end{bmatrix} = (-1) \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ & \ddots & \\ \frac{\partial q_1}{\partial p_{J_A}} & & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{q}_1(\mathbf{p}_t^e) \\ \vdots \\ \tilde{q}_{J_A}(\mathbf{p}_t^e) \end{bmatrix}. \tag{F.2}$$

.

After the merger with firm B, firm A considers the impact of its pricing on the profit of products owned by firm B. The system of FOCs are now given by

$$\begin{aligned} &\begin{bmatrix} \tilde{q}_1(\mathbf{p}_t^e) \\ \vdots \\ \tilde{q}_{J_A}(\mathbf{p}_t^e) \end{bmatrix} + \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ & \ddots & \\ \frac{\partial q_1}{\partial p_{J_A}} & & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix} \begin{bmatrix} p_1 - mc_1 \\ \vdots \\ p_{J_A} - mc_{J_A} \end{bmatrix} \\ &+ \begin{bmatrix} \frac{\partial q_{J_A+1}}{\partial p_1} & \cdots & \frac{\partial q_{J_A+J_B}}{\partial p_1} \\ & \ddots & \\ \frac{\partial q_{J_A+1}}{\partial p_{J_A}} & & \frac{\partial q_{J_A+J_B}}{\partial p_{J_A}} \end{bmatrix} \begin{bmatrix} p_{J_A+1} - mc_{J_A+1} \\ \vdots \\ p_{J_A+J_B} - mc_{J_A+J_B} \end{bmatrix} = 0 \end{aligned}$$

Rewriting this, we define the UPP as

$$\begin{bmatrix} p_1 - (mc_1 + UPP_1) \\ \vdots \\ p_{J_A} - (mc_{J_A} + UPP_{J_A}) \end{bmatrix} = (-1) \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ & \ddots & \\ \frac{\partial q_1}{\partial p_{J_A}} & & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{q}_1(\mathbf{p}_t^e) \\ \vdots \\ \tilde{q}_{J_A}(\mathbf{p}_t^e) \end{bmatrix}, \tag{F.3}$$

where the UPP for firm A is defined as follows:

$$\begin{bmatrix} UPP_1 \\ \vdots \\ UPP_{J_A} \end{bmatrix} = -\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \cdots & \frac{\partial q_{J_A}}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_1}{\partial p_{J_A}} & \cdots & \frac{\partial q_{J_A}}{\partial p_{J_A}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial q_{J_A+1}}{\partial p_1} & \cdots & \frac{\partial q_{J_A+J_B}}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_{J_A+1}}{\partial p_{J_A}} & \cdots & \frac{\partial q_{J_A+J_B}}{\partial p_{J_A}} \end{bmatrix} \begin{bmatrix} p_{J_A+1} - mc_{J_A+1} \\ \vdots \\ p_{J_A+J_B} - mc_{J_A+J_B} \end{bmatrix}. \tag{F.4}$$

Note the vector of UPP is considered as the additional cost for firm A.

G Construction of Rental Price

This appendix explains the details of constructing the rental price. We follow Abe (2023) to construct the rental price of automobiles (See Footnote 5 of the paper). The rental price of an automobile is comprised of three components: (1) depreciation, (2) loan interests, and (3) specific tax. We explain each component in detail below.

First, the depreciation is computed based on the statutory service life of automobiles as stipulated by the National Tax Agency: 6 years for a normal car and 4 years for a minicar:

$$d_{jt} = \begin{cases} \left[(1+\rho_{jt})p_{jt} - ES_{jt}\right]/6 & \text{(Normal Car)} \\ \left[(1+\rho_{jt})p_{jt} - ES_{jt}\right]/4 & \text{(Minicar)} \end{cases}$$

where ρ_{jt} is the rate of the ad-valorem tax, including consumption tax (5% during the sample period) and ES_{jt} is ecocar subsidy.

The second component is the loan interest. Loans are primarily categorized into financial institution loans and dealer loans, with the former typically ranging from approximately 1.5% to 3.0%, and the latter ranging from approximately 6.0% to 9.0%. Here, we assume the loan interest to be 3% for simplicity:

$$l_{jt} = ((1+\rho_{jt})p_{jt} - ES_{jt}) * 0.03$$

In addition to the previous two components, car buyers have to pay specific taxes, that is, weight tax (w_{jt}) and automobile tax (a_{jt}). Since the buyers have to pay the weight tax for the first three years (until the first vehicle inspection) in a lump sum at the time of initial registration of the vehicle, we divide the amount of weight tax paid at the time of purchase by 3:

$$t'_{jt} = a_{jt} + \frac{w_{jt}}{3}$$

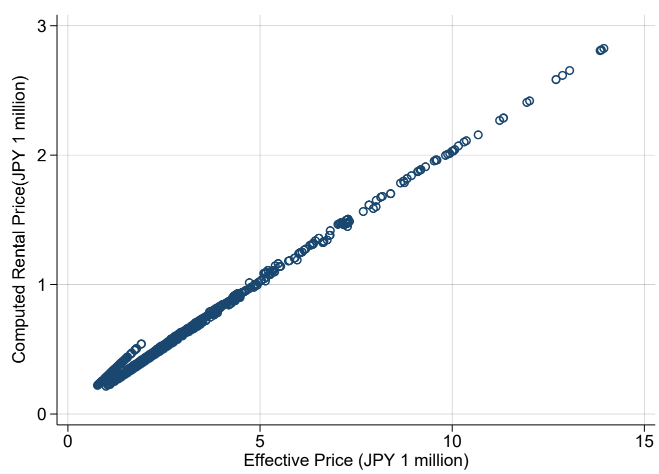
In sum, the rental price that the buyer faces at the time of purchase can be written as the function of the price set by the firm:

$$p^r_{jt} = d_{jt} + l_{jt} + t'_{jt} = \lambda_{jt}(1 + \rho_{jt})p_{jt} - \lambda_{jt} ES_{jt} + t'_{jt}$$

where $\lambda_{jt} = 1/6 + 0.03$ for normal cars and $1/4 + 0.03$ for minicars. Figure A4 shows the relationship between the rental price and effective prices. Given the construction explained above, the relationship is almost linear. The rental price is approximately 20% of the effective price.

Reflecting the construction of rental price, the pass-through rate in this model (See second row in panel A and B of Table 14) is calculated by $PTR_{jt} = \frac{p^{r'}_{jt} - p^r_{jt}}{\lambda_{jt} ES_{jt}}$, where $p^{r'}_{jt}$ indicates the simulated rental price in the counterfactual case without ES.

Figure A4: Relationship Between Rental Price and Effective Price



H Measurement of Compensating Variation

H.1 Overview

We measure changes in consumer welfare associated with a price change using compensating variation (hereafter CV), which is the amount of money a consumer would need to be indifferent to the change. Let the baseline price be $\mathbf{p}$ and the counterfactual price $\mathbf{p}'$. The indirect utility is defined as follows:

$$W(\mathbf{p}, y) = \max_{j \in J_i} V_{ij}, \tag{H.1}$$

where $V_{ij} = v_j(p_j, y) + \epsilon_{ij}$ and ϵ_{ij} has the joint cumulative distribution $F(\epsilon_1, \cdots, \epsilon_{J_t})$. In our application, $v_j(p_j, y) = f(y - p_j) + \beta X_j + \xi_j$ holds.

We denote the individual-level CV using cv, which is defined as

$$W(\mathbf{p}, y) = W(\mathbf{p}', y - cv). \tag{H.2}$$

CV should be interpreted as a random variable because it depends on the idiosyncratic shock $(\epsilon_1, \cdots, \epsilon_{J_t})$. Consequently, we focus on the mean CV $\mathbb{E}(cv)$ as a welfare measure.

If ones assumes the linear utility, measuring CV can be relatively straightforward when using the log-sum formula proposed by Small and Rosen (1981). In our paper, we use the theoretical results produced by Dagsvik and Karlström (2005) to calculate $\mathbb{E}(cv)$.[45] Using their method, the computation of CV reduces to a sum of a one-dimensional integral, which can be easily calculated using numerical methods. Moreover, when the idiosyncratic shock $\boldsymbol{\epsilon}$ follows the i.i.d. Type I extreme-value distribution, the calculation of the integral becomes much simpler.

To explain the method proposed by Dagsvik and Karlström (2005), we first define the random expenditure function $Y(\mathbf{p}, u)$ by using the following equation:

$$u = W(\mathbf{p}, Y(\mathbf{p}, u)). \tag{H.3}$$

The expenditure function $Y(\mathbf{p}, u)$ is interpreted as the income level under which the consumer can achieve the utility level of u when the price vector is $\mathbf{p}$.

[45] Griffith et al. (2018) also use this method to derive the mean CV in their application.

The CV can be defined as

$$cv = y - Y(\mathbf{p}', W(\mathbf{p}, y)). \tag{H.4}$$

Using this equation, the expected CV $\mathbb{E}(cv)$ can be obtained by calculating $\mathbb{E}[Y(\mathbf{p}', W(\mathbf{p}, y))]$.[46] Dagsvik and Karlström (2005) formulates useful theorems to derive the distribution function of the random variable $Y(\mathbf{p}, u)$. Below, we present an overview of this derivation.

H.2 Derivation of the general case

First, we consider the joint distribution of the random expenditure $Y(\mathbf{p}, u)$ and the optimal choice $J(\mathbf{p}, y)$, which is defined by

$$J(\mathbf{p}, y) = \arg\max_{j \in J} v_j(p_j, y) + \epsilon_{ij}.$$

Theorem 3 of Dagsvik and Karlström (2005) derives the formal expression of this joint distribution:

$$\begin{aligned}\mathbb{P}\left(Y(\mathbf{p}', W(\mathbf{p}, y))\right. &> \left. z, J(\mathbf{p}, y) = i\right) \\ &= \begin{cases} \int F_i(u - h_1(p_1, y, p_1', z), \cdots, u - h_J(p_J, y, p_J', z))du & \text{if } 0 < z < y_i(p_i, y, p_i') \\ 0 & \text{if } z \geq y_i(p_i, y, p_i') \end{cases}\end{aligned}$$

where F_i denotes the partial derivative of the cumulative distribution $F(\epsilon_1, \cdots, \epsilon_{J_t})$ with respect to i-th input. $h_j(p_j, y, p_j', z)$ is defined by

$$h_j(p_j, y, p_j', z) \equiv \max\left\{v_j(p_j, y), v_j(p_j', z)\right\}$$

and $y_j(p_j, y, p_j')$ is defined by the following equation.

$$v_j(p_j, y) = v_j(p_j', y_j(p_j, y, p_j')).$$

Intuitively speaking, $y_j(p_j, y, p_j')$ is the income level needed to obtain the utility level of $v_j(p_j, y)$ when the price is p_j'.

[46] This is because by substituting (H.4) to (H.2), we get $W(\mathbf{p}, y) = W(\mathbf{p}', Y(\mathbf{p}', W(\mathbf{p}, y)))$, where the equality must hold by the definition of the expenditure function.

We now derive the marginal distribution of the random expenditure $Y(\mathbf{p}, u)$, which will be used to calculate $\mathbb{E}[Y(\mathbf{p}', W(\mathbf{p}, y))]$. Corollary 2 of Dagsvik and Karlström (2005) has shown that the marginal distribution is derived as follows:[47]

$$\mathbb{P}(Y(\mathbf{p}', W(\mathbf{p}, y)) > z) = \\ \sum_{i \in \mathbf{J}} I_i(p_i, y, p_i', z) \times \int F_i(u - h_1(p_1, y, p_1', z), \cdots, u - h_J(p_J, y, p_J', z))du.$$

where the indicator function $I_i(p_i, y, p_i', z)$ is defined as

$$I_i(p_i, y, p_i', z) = \begin{cases} 1 & \text{if } v_i(p_i, y) > v_i(p_i', z) \\ 0 & \text{otherwise} \end{cases} \tag{H.5}$$

Using the marginal distribution, we now calculate the expectation $\mathbb{E}(Y(\mathbf{p}', W(\mathbf{p}, y)))$ by

$$\begin{aligned} \mathbb{E}(Y(\mathbf{p}', W(\mathbf{p}, y))) &= \int_0^\infty Y(\mathbf{p}', W(\mathbf{p}, y)) \cdot d\mathbb{P}(Y(\mathbf{p}', W(\mathbf{p}, y)) \leq z) \\ &= \int_0^\infty \mathbb{P}(Y(\mathbf{p}', W(\mathbf{p}, y)) > z)dz \qquad \text{(H.6)} \\ &= \sum_{i \in \mathbf{J}} \int_0^{y_i(p_i, y, p_i')} \int F_i(u - h_1(p_1, y, p_1', z), \cdots, u - h_J(p_J, y, p_J', z))dudz \qquad \text{(H.7)} \end{aligned}$$

The second equality uses Lemma 1 of Dagsvik and Karlström (2005).[48]

H.3 Special Case: i.i.d. Type-1 Extreme Value Distribution

When the idiosyncratic shock follows Type-I extreme value distribution, the integral of choice probability has a closed form expression (McFadden, 1981) [49]. Therefore, in this case, the joint

[47] The marginal distribution can be obtained by adding up the joint distribution for goods i, thus satisfying $I_j(p_i, y, p_i', z) = 1$. Note that $z < y_i(p_i, y, p_i')$ is equivalent to $v_j(p_j, y) > v_j(p_j', z)$.

[48] Let G be the cumulative distribution function of a random variable x. Lemma 1 of Dagsvik and Karlström (2005) shows that, for any $\alpha \geq 1$, $\int_0^\infty x^\alpha dG(x) = \alpha \int_0^\infty x^{\alpha-1}(1 - G(x))dx$. We apply this lemma when $\alpha = 1$.

[49] This is a special case of the Generalized Extreme Value (GEV) model in which the choice probability of j-th alternative can be expressed as $P_j = \int_{-\infty}^{+\infty} F_j(v_j + \epsilon_j - v_1, \cdots, v_j + \epsilon_j - v_J)d\epsilon_j = y_j G_j / G$ using some function $G(e^{v_1}, e^{v_2}, \cdots, e^{v_J})$ having the following properties: (i) $G(e^{v_1}, e^{v_2}, \cdots, e^{v_J}) \geq 0$ for all j, (ii) G is linearly homogeneous (i.e. $G(\rho e^{v_1}, \rho e^{v_2}, \cdots, \rho e^{v_J}) = \rho G(e^{v_1}, e^{v_2}, \cdots, e^{v_J})$), (iii) $\lim_{v_k \to \infty} G = +\infty$ for all k, and (iv) n-th order derivative is non-negative if n is odd, and non-positive if n is even. When the error term follows Type-I extreme value distribution, the function G corresponds to $G = \sum_{j \in \mathbf{J}} e^{v_j}$. See McFadden (1981) for details.

distribution of expenditure function and the choice can be rewritten as, for $z < y_i(p_i, y, p'_i)$,

$$\begin{aligned}
&\mathbb{P}(Y(\mathbf{p}', W(\mathbf{p}, y)) > z, J(\mathbf{p}, y) = i) \\
&= \int F_i(u - h_1(p_1, y, p'_1, z), u - h_2(p_2, y, p'_2, z), \cdots, u - h_J(p_J, y, p'_J, z))du \\
&= \frac{\exp(h_i(p_i, y, p'_i, z))}{\sum_{k\in\mathbf{J}} \exp(h_k(p_k, y, p'_k, z))} \\
&= \frac{\exp(v_i(p_i, y))}{\sum_{k\in\mathbf{J}} \exp(\max\{v_k(p'_k, z), v_k(p_k, y)\})}
\end{aligned}$$

The final equality holds by the definition of $h_i(p_i, y, p'_i, z)$ and the restriction of $z < y_i(p_i, y, p'_i)$.[50]

Based on this result, the probability distribution of the expenditure function can be derived as follows:

$$\mathbb{P}(Y(\mathbf{p}', W(\mathbf{p}, y)) > z) = \sum_{i\in\mathbf{J}} I_i(p_i, y, p'_i, z) \frac{\exp(v_i(p, y))}{\sum_{k\in\mathbf{J}} \exp(\max\{v_k(p'_k, z), v_k(p_k, y)\})}$$

Finally, by (H.7), the expectation of expenditure function is

$$\mathbb{E}(Y(\mathbf{p}', W(\mathbf{p}, y)) = \sum_{i\in\mathbf{J}} \int_0^{y_i(p_i, y, p'_i)} \frac{\exp(v_i(p_i, y))}{\sum_{k\in\mathbf{J}} \exp(\max\{v_k(p'_k, z), v_k(p_k, y)\})} dz,$$

which is the final result of Corollary 5 of Dagsvik and Karlström (2005).

As we argued at the beginning of this section, the computation of $\mathbb{E}(cv)$ reduces to the sum of a one-dimensional integral under the standard assumptions (i.e. the error term follows Type-I extreme value distribution). Furthermore, in our counterfactual analysis, because none of the automobile characteristics change except for the price, $y_i(p_i, y, p'_i)$ can be derived by $y_i(p_i, y, p'_i) = y + p'_i - p_i$. Finally, by applying the technique of numerical integration (e.g., Gauss-Legendre quadrature), we can compute the expectation of expenditure function and the expectation of CV.

[50] $z < y_i(p_i, y, p'_i)$ is equivalent to $v_i(p_i, y) > v_i(p'_i, z)$. Thus, $h_i(p_i, y, p'_i, z) = \max\{v_i(p_i, y), v_i(p'_i, z)\} = v_i(p_i, y)$.

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