Lecture 5: Oligopoly Competition

Cabral Chapter 8

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Oligopoly Competition

- We have studied the extreme cases of market structures so far.
  - monopoly
  - perfect competition.

- Most real-world markets are somewhere between the extremes.

- We now study **oligopolistic competition**.
  - Situation in which there are few competitors.
  - Special case: Duopoly (when the number is two)

- Overview
  - Quick review of game theory
  - Bertrand model (price competition)
  - Cournot model (quantity competition)
What is Game Theory

- Game theory studies decision-making when decision makers interact with each other.
  - Your payoff (profit) depends on what others do.
  - To achieve your most favorable outcome, you need to understand how other players behave and how they interact.

- Why important in IO? **Imperfectly-competitive markets!**
  - Imperfect competition (oligopoly): a few firms
  - Perfect competition: Many price-taking firms.
  - Monopoly: only one firm.
Motivating Example

Consider the following news article: *The wall Street Journal, November 16, 1999.*

“In a strategic shift in the United States and Canada, Coca-Cola Co. is . . . gearing up to raise the prices it charges its customers for soft drinks by about 5% . . . The price changes could help boost Coke’s profit . . .

Important to the success of Coke and its bottlers is how Pepsi-Cola . . . responds. The No.2 soft-drink company could well sacrifice some margins to pick up market share on Coke”, some analysts said.

This article highlights the main points in duopoly.

Firm 1 (Coke) is likely to influence Firm 2 (Pepsi)’s profits, and vice versa.

Coke’s decision process should consider what Pepsi is expected to do.
Setup

- There are $N$ players indexed by $i \in \{1, \cdots, N\}$.
- Player $i$ chooses a strategy $s_i$ from the strategy set $S_i$.
- Player $i$'s payoff (utility) is given by
  \[ u_i(s_1, \cdots, s_i, s_{i+1}, \cdots, s_N) \]
- Here, the utility depends on both her strategy and others' strategies.
- We write the payoff as
  \[ u_i(s_i, s_{-i}) \]
  where $s_{-i}$ is the others' strategy
  \[ s_{-i} \equiv \{s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_N\} \]
Nash Equilibrium

▶ The most standard equilibrium concept.

▶ Definition: The strategy profile \( s^* = (s_1^*, s_2^*, \ldots, s_n^*) \) is a Nash equilibrium if

\[
  u_i(s_i^*, s_{-i}^*) \geq u_i(s'_i, s_{-i}^*) \quad \forall s'_i \in S_i \text{ and all players } i.
\]

▶ Intuition: No player has an incentive to deviate from their strategy \( s_i^* \), given all other players strategies \( s_{-i}^* \).
Nash equilibrium = mutual best response

- Strategy $s_i$ is **the best response strategy** (BR) to $s_{-i}$ if

$$u_i(s_i, s_{-i}) \geq u_i(s', s_{-i})$$

for all $s' \in S_i$

- In other words, $s_i$ is the BR to $s_{-i}$ if $s_i$ maximizes your payoff:

$$s_i = \arg\max_{s \in S_i} u_i(s, s_{-i})$$

- Players are mutually playing **best response strategy** to each other in Nash equilibrium.
Example of Normal-Form Games

- **Game 1: Prisoner’s Dilemma**

<table>
<thead>
<tr>
<th></th>
<th>Quiet</th>
<th>Fink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>−1, −1</td>
<td>−3, 0</td>
</tr>
<tr>
<td>Fink</td>
<td>0, −3</td>
<td>−2, −2</td>
</tr>
</tbody>
</table>

- Nash equilibrium: (Fink, Fink)
### Game 2: Coordination Game

<table>
<thead>
<tr>
<th>Firm</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest in computer line</td>
<td>Acquire computer skills</td>
</tr>
<tr>
<td></td>
<td>Do not acquire computer skills</td>
</tr>
<tr>
<td>Maintain old line</td>
<td>0, $-10$</td>
</tr>
</tbody>
</table>

- (Invest, Acquire) is a NE.
- (Maintain old line, Do not acquire) is another NE.
- We could have multiple equilibria.
Bertrand Model

- A model where firms set the prices for a good simultaneously.
- Number of firms: $j = 1, 2$
- Price competition: Each firm sets price $p_j$
- Assume symmetric and constant marginal cost: $MC(q) = c$
- The goods are homogeneous. Market demand $D(p)$
- Consumers buy the good from the firm with the lowest price.
- Static (one shot) game
Denote the quantity firm $i$ sells by $D_i(p_1, p_2)$ as a function of $(p_1, p_2)$.

Since the goods are homogenous, the buyer chooses the cheapest seller.

The sales for firm $i$

$$D_i(p_1, p_2) = \begin{cases} 
D(p_i) & \text{if } p_i < p_j \\
\frac{D(p_i)}{2} & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}$$

Therefore, the profit of firm $i$ can be expressed as

$$\pi_i(p_1, p_2) = (p_i - c) D_i(p_1, p_2).$$

Question: What is the equilibrium price?

Use Nash equilibrium to predict the outcome.
Step 1: Find the best response given the competitor’s price

- Consider firm $i$’s best response given competitor $j$’s price $p_j$.
- We consider 3 cases, depending on the price $p_j$.

- Case 1: $p_j > c$
  - Firm $i$’s profit
    \[
    \pi_i = \begin{cases} 
    D(p_i)(p_i - c) & \text{if } p_i < p_j \\
    \frac{D(p_i)}{2}(p_i - c) & \text{if } p_i = p_j \\
    0 & \text{if } p_i > p_j 
    \end{cases}
    \]
  - Firm $i$ wants to undercut the price by just a small amount.
Case 2: $p_j = c$

- Firm $i$’s profit

$$\pi_i = \begin{cases} 
D(p_i)(p_i - c) < 0 & \text{if } p_i < p_j \\
\frac{D(p_i)}{2}(p_i - c) = 0 & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}$$

- Firm $i$ wants to set $p_i = c$.

Case 3: $p_j < c$

- Firm $i$ earns either 0 or negative profit.
- Therefore, it wants to set $p_i = c$.

The best response function for firm $i$ is

$$p_i^*(p_j) = \begin{cases} 
p_j - \epsilon & \text{if } p_j > c \\
c & \text{if } p_j = c \\
c & \text{if } p_j < c 
\end{cases}$$
Step 2: Nash Equilibrium as Mutual Best Response
Nash equilibrium is given by $p_1 = p_2 = c$.

To examine this, we should see whether both firms have no incentive to deviate from this situation.

1. $p_i < c$ cannot be NE b.c. firm $i$ can weekly increasing profit by raising prices.
2. $p_i > p_j$ and $p_j \geq c$ cannot be NE b.c. firm $i$ has a profitable deviation.
3. $p_i = p_j > c$ cannot be NE since both firms have a profitable deviation.
4. When $p_1 = p_2 = c$, both of them has no profitable deviation.

In Nash equilibrium, both have zero profit!
The model provides a very extreme result.

- Equilibrium price is \( p_1 = p_2 = c \) regardless of the number of firms.
- Both firms obtain zero profits.

This is called **Bertrand Paradox**.

How can we make the model more realistic?
Solution to Bertrand Paradox

1. **Product Differentiation**
   - We assumed homogenous products: Completely same products.
   - The goods are not exactly the same in reality.
     - Example: Pepsi and coke
     - Differentiation by characteristics, brand images, etc...

2. **Dynamic Competition (tacit collusion)**
   - Firms compete in one period only, a situation like “Prisoners’ Dilemma”.
   - In reality, firms compete over some certain period of time.

3. **Capacity Constraints and/or increasing Marginal Cost.**
   - We assumed the marginal cost is constant and no capacity constraint.
   - Capacity constraint matters in practice!
Cournot Model

- Firms compete with **quantity (production) decision.**

- **Setup**
  - Number of firms: \( j = 1, 2 \)
  - Firms set quantity simultaneously. \( q_1 \) and \( q_2 \).
  - Price is determined to clear the demand.
  - Constant marginal cost: \( MC(q) = c \) (same for both firms)
  - The goods are homogeneous: Market demand \( P(Q) \)
  - Static (one shot) game

- The profit of firm \( i \) is

\[
\pi_i(q_1, q_2) = P(q_1 + q_2)q_i - cq_i.
\]

- Use Nash equilibrium to find the equilibrium quantity.
Nash Equilibrium (or Cournot Equilibrium)

The conditions for \((q_1^*, q_2^*)\) to be a Nash equilibrium:

\[
\pi_1(q_1^*, q_2^*) \geq \pi_1(q_1, q_2^*) \quad \text{for any } q_1 \\
\pi_2(q_1^*, q_2^*) \geq \pi_2(q_1^*, q_2) \quad \text{for any } q_2
\]

NE can be found using **best-response functions**.

- Firm \(i\)'s best-response function gives the profit-maximizing choice of output for firm \(i\) as a function of output produced by firm \(j\).

Firm \(i\)'s best-response function is denoted by \(R_i\):

\[
q_i = R_i(q_j).
\]

The NE simultaneously satisfy the best-response functions for both firms:

\[
q_1^* = R_1(q_2^*) \\
q_2^* = R_2(q_1^*)
\]
Consider firm 1’s problem, given $q_2$

$$\max_{q_1} P(q_1 + q_2)q_1 - cq_1$$

The first order condition for $q_1$:

$$P + \frac{\partial P(q_1 + q_2)}{\partial Q}q_1 - c = 0$$

This gives the optimal choice for $q_1$ given $q_2$:

$$q_1 = R_1(q_2).$$

Similarly, we can get the best response for firm 2:

$$q_2 = R_2(q_1)$$
Exercise

Setup

- The market demand is $P(Q) = 100 - Q$ (where $Q = q_1 + q_2$).
- Costs of production are the same among firms 1 and 2: $C(q) = cq$.

What is the Cournot equilibrium?
Exercise cont’d

Let’s first find the best-response function of firm 1.

The profit function of firm 1 is

\[ \pi_1(q_1, q_2) = (100 - q_2 - q_1)q_1 - cq_1. \]

The FOC:

\[ \frac{\partial \pi_1}{\partial q_1} = 100 - q_2 - 2q_1 - c = 0. \]

The best-response function of firm 1:

\[ R_1(q_2) = \frac{100 - q_2 - c}{2}. \]

Similarly, the best-response for firm 2

\[ R_2(q_1) = \frac{100 - q_1 - c}{2}. \]
Best Responses in Figure
Nash Equilibrium requires,

\[ q_1^* = R_1(q_2^*) = \frac{100 - q_2^* - c}{2} \]

\[ q_2^* = R_2(q_1^*) = \frac{100 - q_1^* - c}{2} \].

This yields

\[ q_1^* = \frac{100 - c}{3} \] and \[ q_2^* = \frac{100 - c}{3} \].

The price is

\[ P(Q) = 100 - (q_1 + q_2) = \frac{100 + 2c}{3} \],

and the profit of each firm is

\[ \pi_i(q_1^*, q_2^*) = \frac{(100 - c)^2}{9} \].
General Case with \( N \) firms

- We consider the case with \( N \) firms.

- Setup
  - Number of firms: \( i = 1, 2, \cdots, N \)
  - Cournot competition: Firms set quantities, \( q_i \).
  - Cost Functions: \( C_i(q) \) (can be different among firms.)
  - The goods are homogeneous.
  - (Inverse) market demand \( P(Q) \) where \( Q = \sum_i q_i \).
  - Static (one shot) game
The objective function for firm $i$ is

$$\pi_i(q_i, q_{-i}) = P(Q)q_i - C_i(q_i).$$

$q_{-i}$: a vector of quantity produced by firms other than firm $i$

FOC for firm $i$ is

$$P + \frac{dP}{dQ} q_i - MC_i(q_i) = 0$$

$$P - MC_i = -\frac{dP}{dQ} q_i$$

$$\frac{P - MC_i}{P} = -\frac{dP}{dQ} \frac{q_i}{P} = -\frac{dP}{dQ} \frac{Q}{P} \frac{q_i}{Q}$$

FOC becomes

$$\frac{P - MC_i}{P} = \frac{s_i}{|\epsilon|}.$$  

where $\epsilon$ is the demand elasticity $\frac{dQ}{dP} \frac{P}{Q}$ and $s_i \equiv q_i/Q$ is firm $i$’s market share.
FOC:

\[
\frac{P - MC_i}{P} = \frac{s_i}{|\epsilon|}.
\]

More efficient firms are larger:

- firms with lower \( MC_i \) have larger market share (higher \( s_i \)).

Marker power (or markup) is limited by the demand elasticity

- more elastic demand lowers markup

Cournot markup is always less than monopoly markup

- Monopoly when \( s_i = 1 \).

Suppose that firms are symmetric (\( MC_i \) is same across firms, so that \( q_i \) is the same and \( s_i = \frac{1}{N} \)). If \( N \to \infty \), then \( p \to MC_i \).
Relation with Herfindahl-Hirschman Index ($HHI$)

- A common measure of market concentration:

$$HHI = \sum_{i=1}^{N} s_i^2.$$ 

- $HHI$ varies between 0 (perfect competition) and 1 (monopoly).
- $HHI$ is higher when there are fewer firms or when variations in market share are larger.

- Numerical examples:
  - $(s_1, s_2, s_3) = (40\%, 40\%, 10\%)$
  - $(s_1, s_2, s_3) = (33\%, 33\%, 33\%)$
HHI as a measure of Market Power (Markup)

- If we multiply the both sides of FOC by $s_i$ and take summation, then

\[
\sum_{i=1}^{N} s_i \left( \frac{P - MC_i}{P} \right) = \frac{HHI}{|\epsilon|}
\]

weighted average markup
(industry-wide Lerner index)

- HHI as an “incomplete” measure of market power!

- Markup depends on both HHI and $|\epsilon|$.

- Higher HHI (or concentration) does not always lead to higher markup.
HHI and Antitrust Policy

- HHI as a screening devise in merger review by antitrust authorities.

- Ex: Japanese-FTC does not review a proposed merger if it satisfies
  1. $HHI \leq 0.15$ before merger
  2. $HHI > 0.15$ & $HHI \leq 0.25$ before merger and $\Delta HHI \leq 0.025$
  3. $HHI > 0.25$ before merger and $\Delta HHI < 0.015$.

- We will study horizontal mergers later this course in more detail.
Comparative Statics

- **Comparative Statics**: an exercise to see the equilibrium effect of changes in exogenous conditions.

- Example 1: If the price of chicken falls, how does the price of fried chicken change?

- Example 2: If you have a opportunity to invest in a new cost-reduction technology, how should you decide?
Setup

- Number of firms: $j = 1, 2$

- Constant marginal cost: $MC_i(q) = c_i$ (different for each firm)

- The goods are homogeneous: Market demand $P(Q) = a - bQ$
The best-response function of firm 1 and 2 are,

\[ R_1(q_2) = \frac{a - bq_2 - c_1}{2b}, \]
\[ R_2(q_1) = \frac{a - bq_1 - c_2}{2b}. \]

And the equilibrium quantities, price and profits are

\[ q_1^* = \frac{a - 2c_1 + c_2}{3b} \quad \text{and} \quad q_2^* = \frac{a - 2c_2 + c_1}{3b}, \]
\[ Q^* = \frac{2a - c_1 - c_2}{3b} \quad \text{and} \quad P^* = \frac{a + c_1 + c_2}{3}, \]
\[ \pi_1^* = \frac{1}{b} \left( \frac{a - 2c_1 + c_2}{3} \right)^2 \quad \text{and} \quad \pi_2^* = \frac{1}{b} \left( \frac{a + c_1 - 2c_2}{3} \right)^2. \]
Example 1: Increase in Input Cost

Let’s first assume \( c_1 = c_2 = c \). What happens if \( c \) increases?

\[
\frac{\partial Q^*}{\partial c} = \frac{\partial}{\partial c} \left( \frac{2a - c_1 - c_2}{3b} \right) = -\frac{2}{3b} \quad \text{and} \\
\frac{\partial P^*}{\partial c} = \frac{\partial}{\partial c} \left( \frac{a + c_1 + c_2}{3} \right) = \frac{2}{3}
\]

If marginal cost increases by 40%, equilibrium price will increase by \( \frac{2}{3} \times 40\% = 26.6\% \).

- Only 2/3 of the increase in MC will translate into the increase in price.
- In perfect competition, 100% of the increase in marginal cost will translate into the increase in price.

Note: Here MC is constant, so that the pass-through is 100% under perfect competition. If MC is not constant (i.e., increasing marginal cost), the pass-through is less than 100%.
Benefit of Cost Reduction

▶ Thought experiment:
  ▶ Suppose that you currently produce 30.
  ▶ You have an investment opportunity to reduce your marginal cost by 5.
  ▶ An idea: “By the investment, I can reduce the cost by 150. So, I should invest if the investment cost is less than 150”
  ▶ Does this make sense? No in general!

▶ Difficult question because
  ▶ By reducing the cost, the firm can increase the output.
  ▶ The rival rationally expect that the firm increases output and best-respond to the change in output.
  ▶ Without solving the model, it is impossible to foresee the final outcome.
Benefit of Cost Reduction

Note that

\[
\frac{\partial q_1^*}{\partial c_1} = -\frac{2}{3b}, \quad \frac{\partial q_2^*}{\partial c_1} = \frac{1}{3b}, \\
\frac{\partial P^*}{\partial c_1} = \frac{1}{3b}, \quad \frac{\partial Q^*}{\partial c_1} = -\frac{1}{3b}, \\
\frac{\partial \pi_1^*}{\partial c_1} = \frac{1}{b^2} \left( \frac{a - 2c_1 + c_2}{3} \right) \frac{-2}{3} = -\frac{4}{3} q_1^*
\]

- The decrease in firm 1’s marginal cost increases the output.
- Firm 2 rationally expect that and reduces its output.
- Total output increases and price decreases.
- In equilibrium, the profit increases by $\frac{4}{3} q_1^*$.
- If current output level is 30 and you can reduce your cost by 5, the value of such investment is more than 150 (approximately 200).
Bertrand Competition with Capacity Decisions (Kreps and Scheinkman 1983)

Consider the following two-stage model.

- Two firms: $i = 1, 2$
- Two periods: $t = 1, 2$
  
  **Period 1** Firms decide their capacity, $k_i$, simultaneously.
  
  **Period 2** Firms set the price simultaneously.

- To build their capacity, firms need to pay cost, $c(k) = ck$.
- The production in period 2 is costless. However, firms can’t produce more than their capacity.
- The goods are homogeneous: Market demand $P(Q)$
- Consumers buy the good with lower price.

In the subgame perfect Nash equilibrium, the solution to this game is the same as the Nash equilibrium in Cournot model with marginal cost $c$. 
Cournot vs Bertrand

- Which model provides a better description depends on the feature of industries.

- Case 1: If firms must make capacity decision which is hard to adjust in the short run, Cournot is a better approximation.
  - The two stage model of capacity and price decisions provides the same prediction as Cournot (see in the previous slide).
  - ex. Cement, steel, etc..

- Case 2: Firms can adjust the output easily, Bertrand is a better approximation.
  - In a Bertrand model, firms set prices and receives demand based on those prices.
  - This implicitly assumes that firms can produce an output exactly equal to the quantity demanded.
  - ex. software, retail gasoline station.